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A Condorcet Jury Theorem for Unknown Juror Competence

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1 The Condorcet formula

A jury of n members is trying Jack for murder. A number n_1 vote that Jack is guilty, H^1, while the remaining n_0 members vote that he is innocent, H^0. Jurors are characterised by

$$p(V_i^j|H^j \cap V_{i'}^{j'}) = p(V_i^j|H^j) = q_j > 1/2.$$

If H^j is in fact true, the event that jury member i votes for H^j, denoted V_i^j, has some fixed chance q_j, the competence. We assume that the competences are greater than one half.

Condorcet jury theorem

We can now introduce Condorcet's jury theorem. Say that H^1 is true. For an ever larger jury size n, consider the relative frequency of voters in favour,

$$f_1 = \frac{n_1}{n} = 1 - f_0.$$

By the law of large numbers, the probability that f_1 differs from q_1 tends to 0. Because $q_1 > 1/2$, we have:

Assuming H_1, the probability of a correct majority vote $\Delta = n_1 - n_0 > 0$ tends to 1 in the limit.

Inverse Condorcet theorem

Rather than calculating the probability of a majority of votes given the truth of H^j, we might ask for the probability of the hypothesis H^j given some majority of votes:

$$p(H^1|V_{n\Delta}) = \frac{q_1^{n_1}(1-q_1)^{n_0}p(H^1)}{p(V_{n\Delta})}.$$

Here $V_{n\Delta}$ is the vote of the entire jury. With this we can derive an inverse version of Condorcet's theorem:

> *Let the jury size n go to infinity and assume a fixed relative majority larger than ½, then the odds for H^1 will tend to infinity.*

Condorcet formula

Under the idealising assumptions that

- the priors of H^0 and H^1 are equal, $p(H^1) = p(H^0)$, and that
- the competences of jury members on H^0 and H^1 are equal, $q_0 = q_1 = q$,

List (BJPS 2004) derives the following posterior odds:

$$\frac{p(H^1|V_{n\Delta})}{p(H^0|V_{n\Delta})} = \left(\frac{q}{1-q}\right)^{\Delta}.$$

It depends only on the absolute margin of the jury vote, and not on the number of jurors.

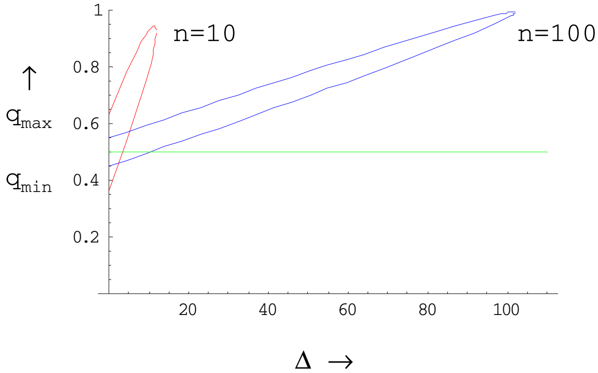
2 Counterintuitive consequences

List emphasises the significance of the absolute margin for jury votes. But the sole dependence on Δ is rather puzzling. Which of the following two juries do you prefer?

	Small jury	Large jury
Number of jurors	10	100
Number in favour (n_1)	10	56
Number against (n_0)	0	44
Absolute margin (Δ)	10	12

A classical statistical analysis

A 95% confidence interval for juror competence *q* shows that the votes are a freak accident, or otherwise that the jurors from the smaller jury are more competent.



Learning competence

The vote of the jury somehow reveals the competence of the jury, and this can be used in choosing between jury verdicts. We bypass a number of alternative explanations:

- Without the assumption of symmetric competence, the posterior odds do depend on the jury size. But they do not do so in the relevant way.
- A unanimous vote may well result from mindless group-think rather than high competence.
- Alternatively, as Bovens and Hartmann (2004) argue, the coherence of jurors might indicate the veracity of the jury verdict.

3 A model using unknown competences

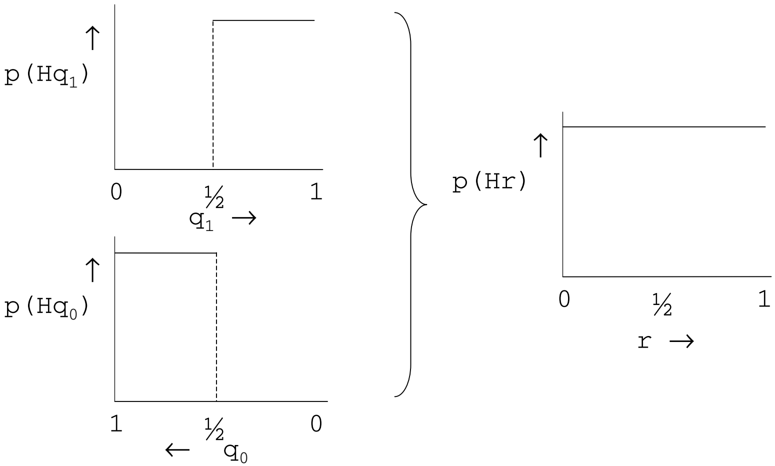
We split the hypotheses H^0 and H^1 up into H_{q_0} and H_{q_1} respectively. The hypotheses H^j each consist of a range of statistical hypotheses:

$$p(H^0) = \int_{1/2}^{1} p(H_{q_0})\, dq_0 \qquad p(H^1) = \int_{1/2}^{1} p(H_{q_1})\, dq_1 \,.$$

The hypotheses H_{q_j} concern the competences $p(V^i_j | H_{q_j} \cap V^{i'}_{j'}) = q_j$. We assume that the prior is equal and uniform over $(1/2, 1)$, for both q_0 and q_1.

Transforming the problem

We can turn this into a well-known statistical problem by a suitable translation of the parameters q_j to a single $r \in [0, 1]$.



Posterior for the hypothesis

We model the impact of the jury vote on the probability assignments over q_0 and q_1 by modelling its impact on the probability assignment over r. The posterior over H_r is a Beta distribution,

$$p(H_r|V_{n\Delta}) = \frac{(n+1)!}{n_0!n_1!} r^{n_1}(1-r)^{n_0}.$$

The posterior probability of the hypotheses H^0 is:

$$p(H^0|V_{n\Delta}) = \frac{(n+1)!}{n_0!n_1!} \int_0^{1/2} r^{n_1}(1-r)^{n_0}\, dr$$

4 Analytic and numerical results

We retain an important consequence of the Condorcet formula. On the assumption that $\Delta = n_1 - n_0 > 0$, we have

$$p(H^1|V_{n\Delta}) > p(H^0|V_{n\Delta}).$$

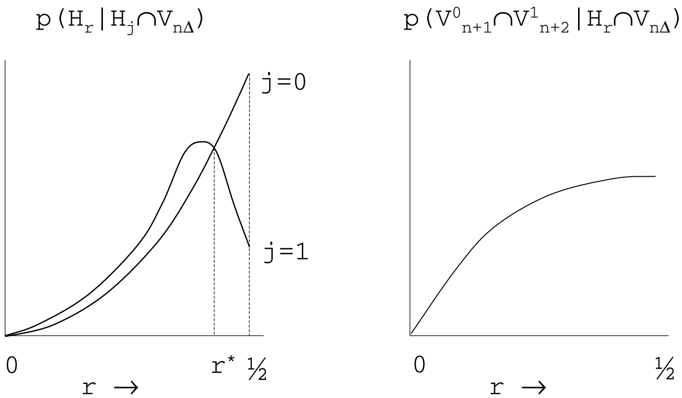
But we can also show that

$$\frac{p(H^1|V_{n+2,\Delta})}{p(H^0|V_{n+2,\Delta})} < \frac{p(H^1|V_{n\Delta})}{p(H^0|V_{n\Delta})}.$$

This repairs the counterintuitive choice between the two juries.

Proof of inequality

Given the likelihoods $r(1-r)$ for H_r, the marginal likelihood of the hypothesis H^0 is larger because most of the mass lies close to $r = ½$.



Limiting behaviour

For constant Δ, we find the asymptotic behaviour

$$\lim_{n\to\infty} p(H^0|V_{n\Delta}) = \frac{1}{2}.$$

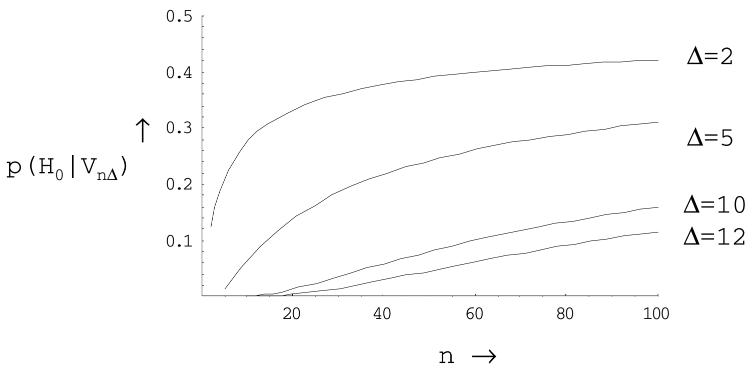
For constant fractional majority, $f = \Delta/n > 0$, we have

$$\lim_{n\to\infty} p(H^0|V_{n,nf}) = 0.$$

In fact the increase in Δ need not be linear in n. It is enough if Δ increases more quickly than $\sqrt{n}$.

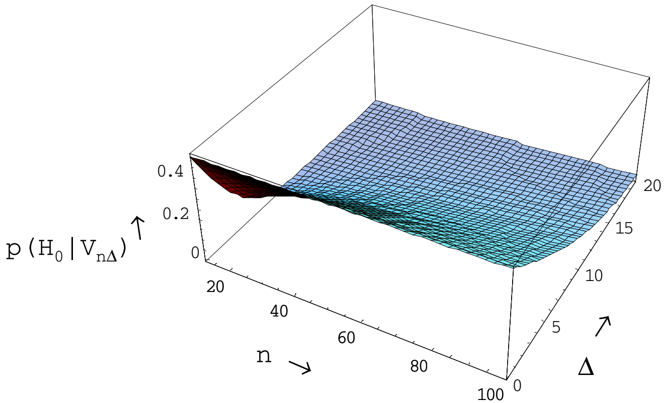
Dependence on jury size

These results are in accordance with the aforementioned intuitions on the relation between jury votes and the hypothesis voted over.



Dependence on jury size and margin

For fixed jury size n, the probability of H^0 decreases with increasing majority size Δ. And for fixed Δ and increasing n, the probability of H^0 increases towards ½.



5 Conclusions

In the model with competence learning we have:

- The probability that the jury majority verdict is incorrect is monotonically increasing in the jury size n, if the absolute margin Δ remains constant.

- The probability that the jury majority verdict is incorrect tends to one-half as n tends to infinity, if Δ remains constant in this limit.

- The probability that the jury majority verdict is incorrect tends to zero as n tends to infinity, if the fractional majority, $f = \Delta/n$, tends to a nonzero constant in this limit.

Important consequences

For the discussion on voting rules, two consequences of this must be given extra emphasis.

- The exclusive dependence on the absolute margin seems to be an artefact of idealising assumptions, and not something inherent to real jury verdicts.
- Both the normal Condorcet jury theorem and the converse Condorcet jury theorem for posterior odds remain valid in the new model.

Hence, against List (2004), we insist on the significance of the relative margin.

Further research

Some suggestions on how to develop the results of the present paper:

- It is important for the practical applicability of Condorcet-style results to relax the assumption on the independence of the jurors (Bovens and Hartmann 2004).
- An entirely different line of research concerns the possible variation of competences within the jury (Dietrich, unpublished).
- Much can be gained by applying the present insights to the discussion over the coherence measures proposed in Bovens and Hartmann (2004).

Thanks!

This talk and the paper on which it is based are both available at http://www.philos.rug.nl/~romeyn.For comments and questions, email j.w.romeijn@rug.nl.