

What if... So what?
Rotterdam 2007

Updating on Conditionals



Igor Douven
Faculty of Philosophy
University of Leuven

Jan-Willem Romeijn
Faculty of Philosophy
University of Groningen

Contents

1	Judy Benjamin	3
2	Rethinking the meaning of conditionals	6
3	Alternative update mechanisms	8
4	A new semantics for conditionals?	12
5	Summary and conclusion	14

1 Judy Benjamin

In van Fraassen's [1989:342 ff], Judy Benjamin is dropped in an area divided into Red (R) and Blue ($\neg R$) and into Second Company (S) and Headquarters ($\neg S$) sections. Initially she assigns equal probability to all quadrants Q_i .

	S	$\neg S$
R	Q_1	Q_2
$\neg R$	Q_3	Q_4

She learns that if she is in Red territory, the odds are 3 : 1 that she is in Headquarters area. How probable is it now that she is in Blue territory?

Cross-entropy minimization

The information imposes a specific constraint on the probability assignment over the segments Q_i . Using a cross-entropy distance function between probability assignments,

$$\Delta(P, P_{old}) = \sum_i P(Q_i) \log \frac{P(Q_i)}{P_{old}(Q_i)},$$

we can look for the closest new probability assignment that satisfies the constraint:

$$\Gamma = \left\{ P : \frac{P(Q_1)}{P(Q_2)} = 3 \right\}, \quad P_{new} = \{ P \in \Gamma : \Delta(P, P_{old}) \text{ minimal} \}.$$

Conditional as material implication

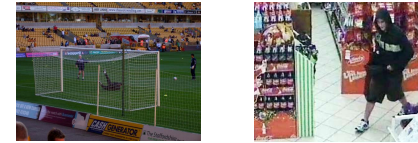
Surprisingly, if we determine the new probability in this way, the probability of being in Blue increases!

	S	$\neg S$
R	Q_1	Q_2
$\neg R$	Q_3	Q_4

Van Fraassen explains this by reference to the limiting case. If Judy learns "If in Red, then in Headquarters, period", the increase in the probability of Blue is a matter of course.

2 Rethinking the meaning of conditionals

We can object to this line of argument for a number of reasons. Consider these stories on a football match and a robbery.



Revised updating on conditionals

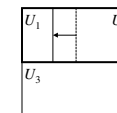
These examples suggest that learning a conditional by updating on its truth conditions misses out on the context dependent implications, i.e. the assertability conditions of conditionals.

- If conditionals have truth conditions: by asserting a conditional we convey more than truth-conditional content. They also have pragmatic implicatures, arguably of a probabilistic nature (Douven 2008).
- If conditionals do not have truth conditions: why take the material implication as a limiting case to begin with?

Below we propose a revised version of updating on a conditional, in which the context dependent implications can be incorporated.

3 Alternative update mechanisms

Say that we learn "If R , then the odds for $\neg S : S$ are $s_0 : s_1$ " and that we do not want to adapt our degree of belief $p(R) = \rho$.



We can achieve this by applying Jeffrey conditionalisation to the partition of events $\mathcal{U} = \{U_1, U_2, U_3\} = \{R \wedge \neg S, R \wedge S, \neg R\}$ and the associated odds, $s_0 : s_1 : (1-\rho)/\rho(s_0 + s_1)$.

A more general update

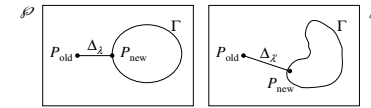
To accommodate differing trade-offs between antecedent and consequent, we may supplement the distance minimisation procedure with a weight function $\lambda_i > 0$,

$$\Delta_\lambda(P, P_{old}) = \sum_{i=1}^4 \lambda_i P(Q_i) \log \frac{P(Q_i)}{P_{old}(Q_i)}$$

The higher λ_i , the more reluctance to change the probability $P(Q_i)$. This idea can be generalised to much more complicated conditional statements. The above application of Jeffrey conditioning is a limiting case.

Epistemic entrenchment

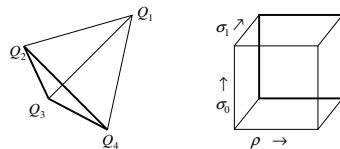
We can model any trade-off between adapting the probability of the antecedent and the consequent by varying the λ_i .



The values of the λ_i express epistemic entrenchment. Or in terms of Lewis' views on conditionals, they determine what is the "closest possible world".

Credal sets

We can also model the update by means of so-called credal sets, sets of possible probability assignments over the events, possibly supplemented with a second-order probability.



Learning a conditional may be modelled as an operation on a suitably parameterised credal set, e.g. by restricting the parameter $\theta_1 = p(Q_1)$ to zero, or $\sigma_1 = P(S|R)$ to one.

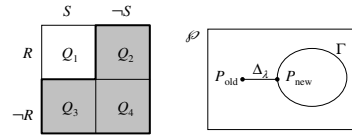
4 A new semantics for conditionals?

The truth conditions of a conditional, if they exist, may be captured in a possible worlds semantics. But on the above models, learning a conditional sentence cannot be understood in the same way.

- When using cross-entropy distance minimisation, the old probability function is not always a convex combination of the possible new probability functions: non-conglomerability.
- Probability assignments cannot be captured straightforwardly in the possible worlds semantics. If anything, operations on credal sets are updates on so-called tail events, events at infinity.

Meaning is epistemic impact

The meaning of a statement is traditionally given by its truth conditions, which also determine what we learn if we accept it as true.



If learning a conditional is modelled in one of the ways proposed in the foregoing, the two come apart. The meaning of a conditional may then be understood in terms of the impact that learning a conditional has on a belief state (Veltman 1996).

5 Summary and conclusion

- We have reason to doubt that learning a conditional is the same as updating on its truth conditions.
- There are various ways of modelling the learning of a conditional by probabilistic dynamics.
- In these models of learning conditionals, their meaning is tied up with their epistemic impact rather than their truth conditions.

Thanks!

This talk will be available at <http://www.philos.rug.nl/~romejn>. For comments and questions, email j.w.romeijn@rug.nl.