Public Good Games on Dynamic Networks: Does Network Topology Matter?

Kai Spiekermann
University of Warwick

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Outline

Cooperation and Spatial Structure

The evolution of cooperation:

- Axelrod's [1984] tournaments: iterated 2-person games
- Skyrms [1996]: Positive correlation of strategies for successful cooperation
- Harder problems: n-person public good games, anonymous contributions

The importance of spatial structure:

- Regulates who interacts with whom and how often
- Provides opportunity for correlation of strategies (clustering)

Cooperation and Spatial Structure (ctd.)

Relevance for Political Philosophy:

- Cooperation fundamental to understand: peace, social norms, «altruism», etc.
- The threat of the free-rider and how to deal with it [Gauthier, 1986, Danielson, 1992]
- Social structure: Social exclusion as a possible sanction
- Group identity, Ingroup-outgroup patterns
- Social justice as an enterprise for mutual benefit?

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PGG on Dynamic Networks

The Idea (informally):

- Agents are either strict cooperators or strict defectors
- Agents are vertices on a graph
- In each round:
  - [game stage] agents play a public good game with their associated group
  - [edge deletion stage] agents can choose to delete one edge to one direct neighbour
  - [edge replacement stage] deleted edges are replaced by new edges
- Run computational simulations and look at payoffs and evolution of network structure

PGG on Dynamic Networks (ctd.)

Game stage

- Each player initiates public good game with associated group H_i (if there are neighbours)
- Each cooperator contributes 1 ($c = 1$), each defector 0 ($c = 0$).
- Payoff function for agent m for the game initiated by i:

$$p_m^i = \begin{cases} s(m,i)\left(r\frac{\sum\limits_{k \in H_i} c_k}{|H_i|} - c_m\right) & \text{for } |H_i| \geq 2 \\ 0 & \text{for } |H_i| < 2. \end{cases}$$

- r is the profit made on public good, assume $r = 1.5$.
- $s(m,i)$: number of edges between m and i. $s(m,i) = 1$ if $m = i$.

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Edge Deletion Stage - Preliminary Considerations

- Agents are boundedly rational:
  - Remember only results from last game stage
  - Use a simple heuristic to decide which edge to delete
- Information available is limited:
  - Contributions are anonymous
  - Agents *do* know how many agents participate in games they play
- A sensible edge deletion strategy? (aim: play with cooperators, avoid defectors)

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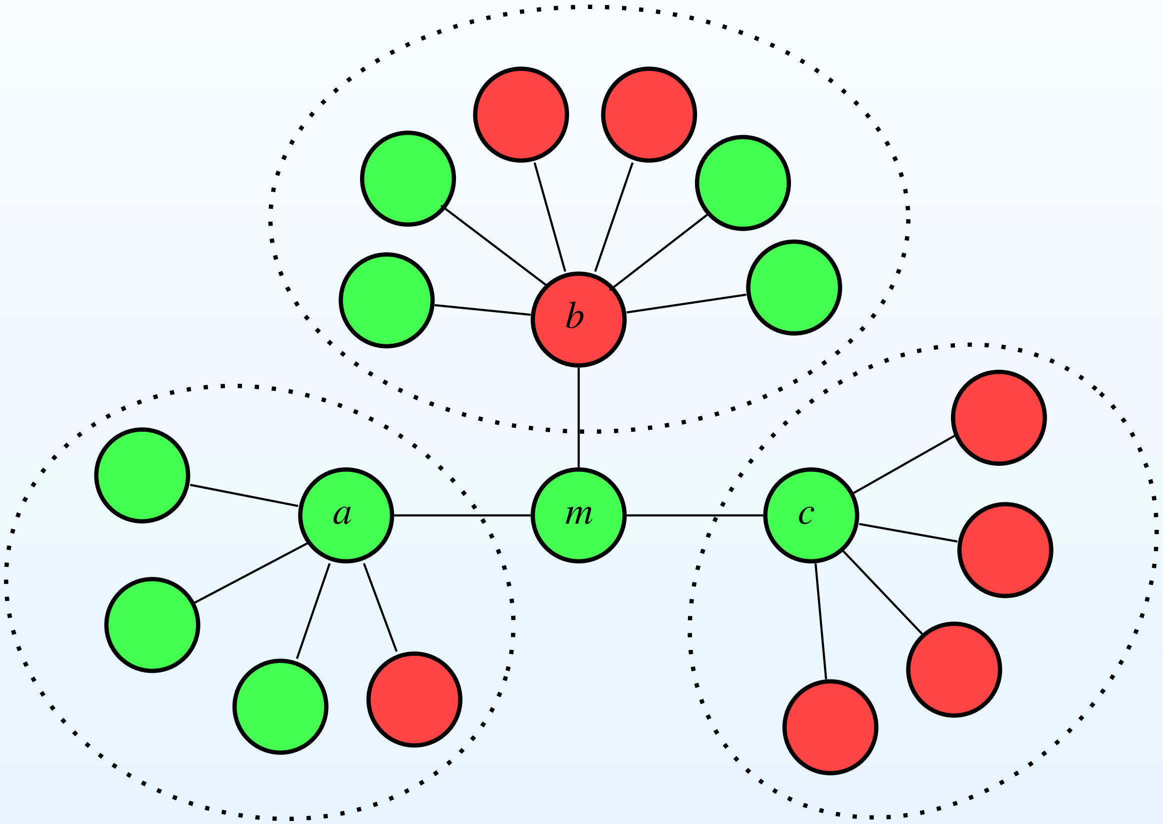
Edge Deletion Stage - An Example



- m plays 4 games
- m knows the defection rate in each game played

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Edge Deletion Stage - Heuristic

- Is there at least one defector in m's neighbourhood? If no, no deletion. If yes:
  - Determine rate of defection in the last game stage in all neighbours' associated groups, excluding m.
  - Delete edge to neighbour with highest rate of defection, break draws by random choice.

Motivation:

- Detach from neighbourhoods with high defection
- First and second degree neighbours matter
- In the example: m deletes edge to c

Procedure: Call all agents in random order to delete edge

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Edge Replacement Stage

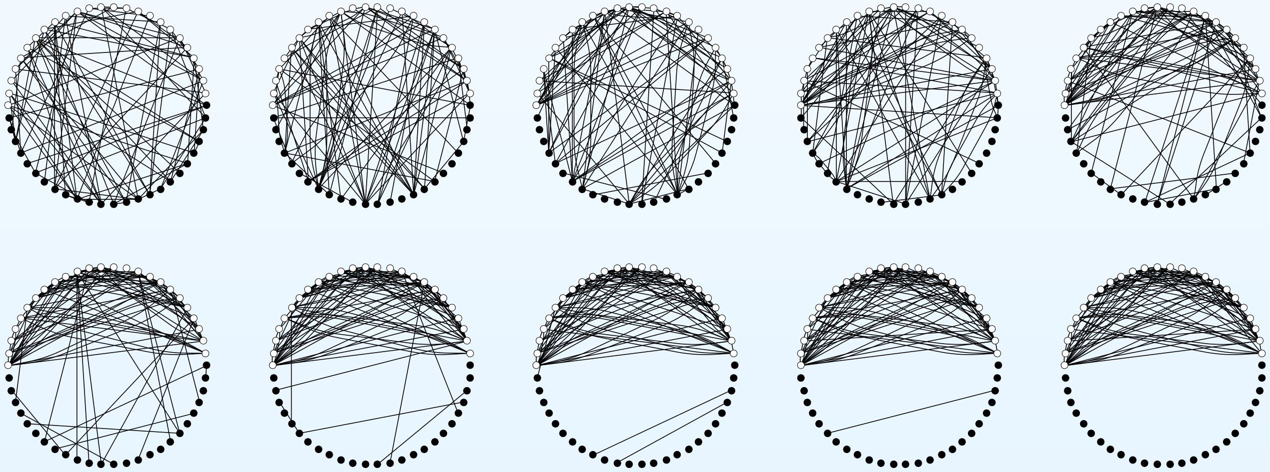
- Replace number of deleted edge (keep number of edges constant)
- Several replacement rules conceivable
- Baseline simulation: Random edge replacement
  - Select two non-identical vertices and draw edge
  - Multiple edges allowed

Summary:

1. Game stage: All agents play PG with associated group
2. Edge deletion stage: according to heuristic
3. Edge replacement stage

Baseline Results Random Nets

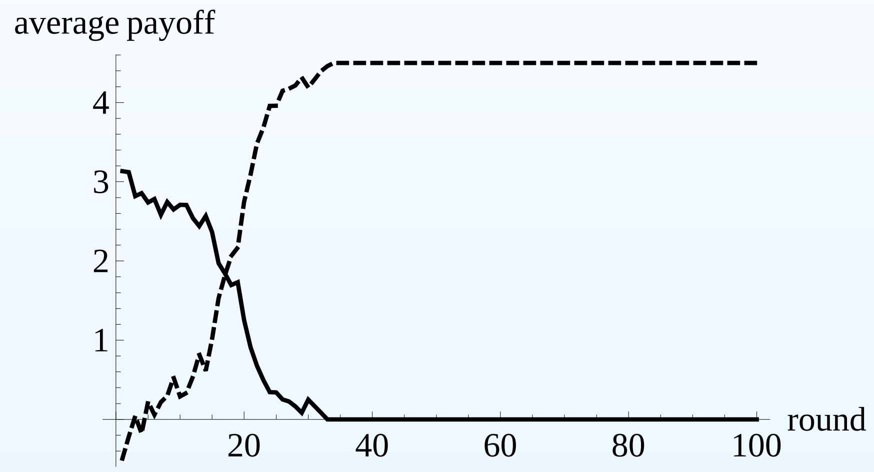
- Computational Simulations to explore model's behaviour
- 50 agents, 25 cooperators, 25 defectors, 100 edges
- Start with random network (different initial topologies below)



(initial topology, and after round 4, 8, 12, 16, . . . ,36)

Baseline Results Random Nets (ctd.)

Average payoff per round for cooperators (dashed) and defectors (solid):



- With random edges, cooperators are exploited, defectors free-ride
- Over time, clustering of cooperators improves payoffs for cooperators
- Isolation of defectors leads to payoff 0

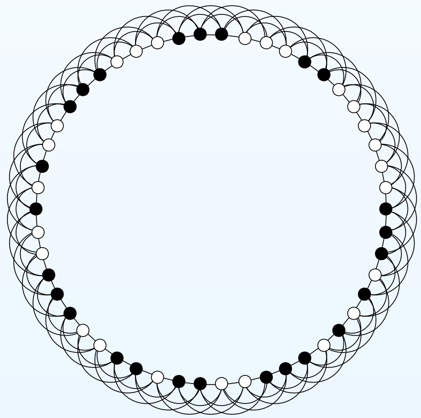
Baseline Results Random Nets (ctd.)

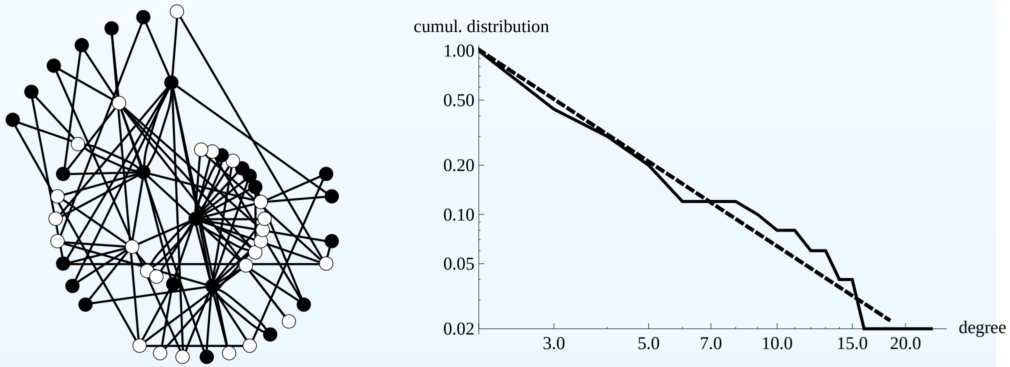
coop (x)	def (y)	edges (k)	rounds	simu-lations	% stable state	payoff coop	payoff def
25	25	100	100	100	100	3.66	0.53
40	10	100	100	100	100	2.92	0.14
10	40	100	100	100	0	-1.81	1.60
10	40	100	1000	100	32	0.60	1.34
100	100	400	100	20	100	3.80	0.44
25	25	200	100	100	47	2.98	3.67
25	25	200	1000	100	100	7.56	0.62

- results stable w.r.t. parameter variation
- Exceptions:
    - small rate of cooperators
    - network with high connectivity

Different Initial Topologies

Effect of different initial topologies: 1D-grid (circulant graph), scale-free graph?





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- (Almost) no effect!
- As edge replacement is unchanged, network converges to similar structure, independent from initial topology

Different Edge Replacement Rules

Different edge replacement rules:

- Local attachment
  - Connect only to «close» neighbours
  - Models situation where interaction is limited by distance
- Preferential attachment
  - It is more likely to connect to agents with many links than to agents with few links
  - Models that highly connected agents are often more likely to meet new agents

Different Edge Replacement Rules (ctd.)

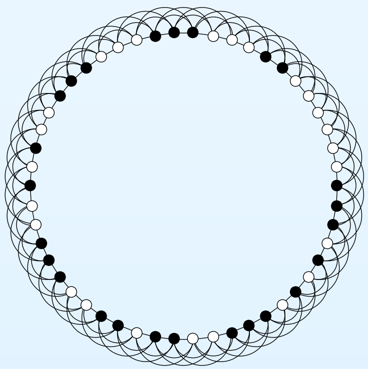
- Anti-preferential attachment
  - It is more likely to connect to agents with few links than to agents with many links
  - Models «crowding» and «search motivation»:
    - Agents with many links are less likely to look for/accept more links
    - Agents with few links have a stronger desire or more time/resources to create links

Conduct computer simulations to explore effects of different edge replacement rules.

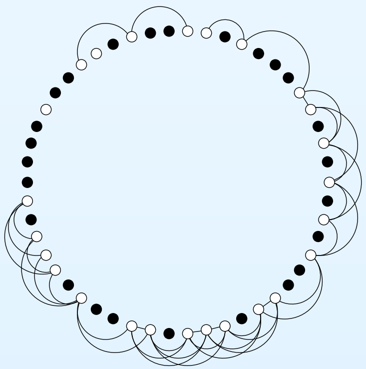
Different Edge Replacement Rules (ctd.)

Local attachment

- start with circulant graph
- Define domain of allowed replacement links as links with distance $2q$ or lower
- Vertices 1,2,...,n are ordered, such that vertex i can connect to $\{i-q, i-q+1, \ldots, i+q-1, i+q\}$.
- Use wrapping, so that vertex 1 and n are neighbours



100 rounds $\Longrightarrow$



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Different Edge Replacement Rules (ctd.)

Preferential Attachment

- Adapting Barabási and Albert [1999], new edges are chosen with a weighted two element random sample
- d: degree of a vertex
- Weight for random sampling: $d+1$
- Probability of drawing a new edge between a and b when replacing one edge is:

$$P(\{a,b\}) = \frac{d_a+1}{\sum_j (d_j+1)} \cdot \frac{d_b+1}{\sum_{j,\, j\neq a} (d_j+1)} + \frac{d_b+1}{\sum_j (d_j+1)} \cdot \frac{d_a+1}{\sum_{j,\, j\neq b} (d_j+1)}.$$

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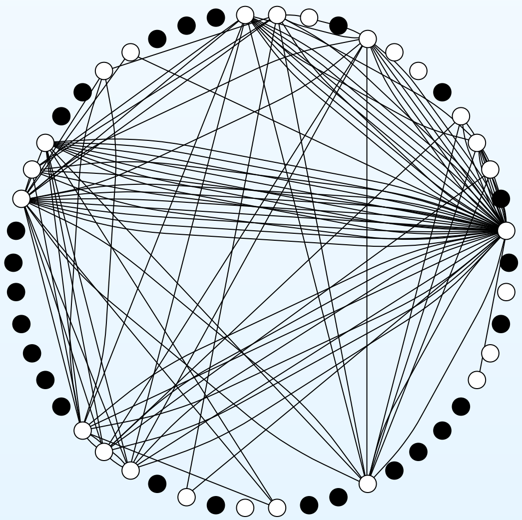
Different Edge Replacement Rules (ctd.)

- Initial network: Scale-free network, 50 vertices (25 cooperators, 25 defectors), 97 edges
- 100 rounds with preferential attachment edge replacement



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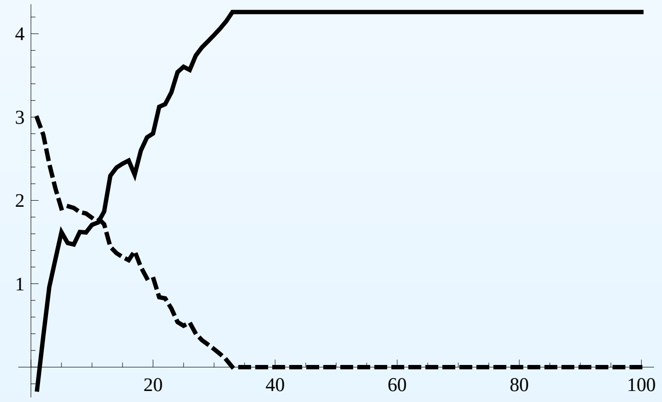

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Different Edge Replacement Rules (ctd.)

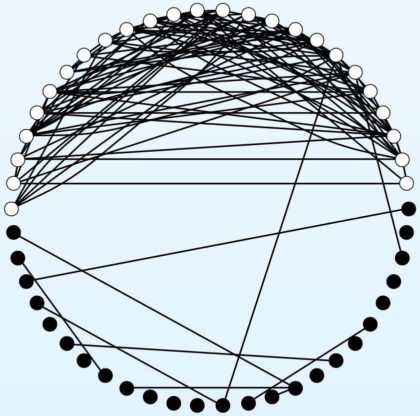
coop (x)	def (y)	initial network	edge replace	edges (k)	rounds	simul-ations	% stable state	payoff coop	payoff def
25	25	PA	random	97	100	100	100	3.67	0.47
25	25	PA	PA	97	100	100	100	3.77	0.37
25	25	PA	PA	190	100	100	79	4.78	2.06
25	25	random	random	190	100	100	49	2.71	3.57
25	25	PA	PA	279	200	100	90	7.83	2.42
25	25	random	random	279	200	100	35	3.30	5.31
100	100	PA	PA	397	100	20	100	3.83	0.39

- cooperators more successful with preferential edge replacement
- In particular for networks with high connectivity

Different Edge Replacement Rules (ctd.)

Anti-Preferential Edge Replacement

- new edges are chosen by two element random sample with weights $1/(d+1)$
- start with random graph, 50 vertices (25 cooperators, 25 defectors), 100 edges
- anti-preferential edge replacement delays separation of cooperators and defectors, one typical result after 100 rounds:



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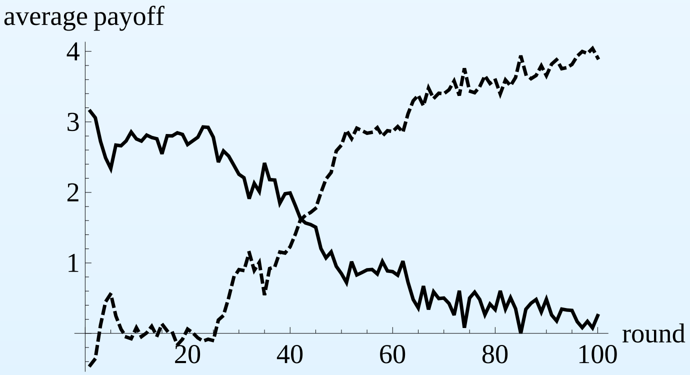

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Conclusion

- Cooperators succeed in clustering, even in anonymous PG games if they can shape the network of interaction
- Different initial graphs have little or no influence on the results
- Edge replacement rules do, because they determine the dynamic topology of the graph
- Preferential attachment edge replacement leads to faster separation
- Possible explanation: Clustering is promoted by preferential attachment
- This conjecture is underscored by the result from anti-preferential edge replacement:
- Anti-preferential edge replacement delays separation of cooperators and defectors
- More research needed to understand games on graphs

Ref

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