The premises of Condorcet's Jury Theorem are not simultaneously justified

by

Franz Dietrich

(Maastricht Univ. & LSE)

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The classic Condorcet Jury Theorem (CJT): informally

Binary decision problem

Exactly one alternative: objectively correct

Goal (of everyone): find correct alternative

Voting rule: majority voting

Let the n. of voters n increase (add more and more voters)

CJT: Under the premises of

- **independence:** the votes are probabilistically independent (in a suitable sense)
- **competence:** each voter votes for the correct alternative with probability $>1/2$, where this probability is the same across voters (homogeneity)

the prob. of a correct majority decision

(a) increases in the group size (if it is odd, to avoid ties)

(b) tends to 1 as $n \to \infty$

–> an optimistic conclusion;

–> if premises justified, good news for epistemic democracy

The problem with the premises (in short)

(i) Whether a premise (independence or competence) is justified depends on the notion of probability considered
(ii) None of the notions renders both premises simultaneously justified.
–> Under the perhaps most interesting notions, the independence assumption should be weakened, but competence might be justified

My critique of the premises focuses on the (asymptotic) part (b)
–> but it essentially extends also to part (a)
–> with one $\neq$: part (a)'s conclusion might be "rescued" by deriving it from more appropriate premises

The (confusion of the) literature on CJT's premises (1)

Most arguments made in the lit. for or against some premise are correct under the author's notion of uncertainty, and incorrect under other notions.

Group deliberation prior to voting is often viewed
(i) as undermining independence (Rawls 1971, Grofman, Owen, and Feld 1983, Ladha 1992, 1995, Dietrich and List 2004),
(ii) or as *not* undermining independence provided voters are isolated once it comes to voting (Waldron in Estlund et al. 1989, Estlund 2007).
–> (ii) is correct if the decision problem (including specific circumstances) is fixed, the former if the problem is variable.

The (confusion of the) literature on CJT's premises (2)

Shared information (Lindley 1985, Dietrich and List 2004), **opinion leaders**, or other influences (Nitzan and Paroush 1984, 1985, Owen 1986, Boland 1989, Boland, Proschan, and Tong 1989, Estlund 1994) are often taken to induce correlations
–> correct in the variable-problem setting

My goal: clarify a discussion that seems to suffers from some confusion
My analysis overlaps with existing arguments (e.g. Ladha 1993, Dietrich and List 2004)

The technical CJT literature

–> A large literature

–> many technical refinements of the CJT

- e.g. (besides the just cited) Young (1988), Berend and Paroush (1998), List and Goodin (2001), Bovens and Rabinowicz (2006), ...
  –> I stick to the classic CJT, but the arguments can be extended
- a recent game-theoretic literature: investigates when sincere or informative voting is a rational strategy (Austen-Smith and Banks 1996, Feddersen and Pesendorfer 1997, Conghlan 2000, Koriyama and Szentes 1995)
  –> I assume sincere/informative voting
  (–> if time allows I'll speak a bit about strategic voting)

Voters

Group of individuals (judges, citizens, experts, etc.), labelled $i = 1, 2, ..., n$, where n (≥ 2) is the group size.
We allow the group size n to vary.
–> Think of the group of size n as containing the first n individuals of an infinite sequence of potential voters $i = 1, 2, 3, ...$
Two alternatives, labelled 0 and 1
–> e.g. 'acquit' or 'convict' the defendant.
One of the alternatives is factually *correct* and the other one *incorrect*.
–> e.g. 'convict' is correct IFF the defendant has commit the crime
Each voter votes for the alternative he believes to be correct (sincere voting)

Decision problems

I define a **decision problem** as the task of *finding a certain correct alternative* x *(0 or 1) under certain circumstances* c.

Thus a decision problem is characterised by two components:

- The **correct alternative** or **state** $x \in \{0, 1\}$.
- The **circumstances** c individuals face.
  - Some are of an evidential kind,
  - others of a non-evidential kind.

Circumstances (1)

Evidential circumstances:

- –> generally observable facts supporting the correctness of 0 or 1, including
  (i) the specific nature of alternatives 0 and 1 (Is it 'acquit Mr. Smith' vs. 'convict him to 7 years prison'? Or 'acquit him' vs. 'convict him to 3 years prison'?)
  (ii) several observable events (the process of group deliberation, fingerprints, a witness report, the defendant's facial expression during the trial, relevant statistical data, ...)

Circumstances (2)

Non-evidential circumstances: events that carry no information on which alternative is correct but may causally affect voting behaviour
–> e.g. room temperature while voting, whether birds are singing (which might induce optimistic belief in the defendant's innocence).
–> One might regard non-evidential circumstances as factors that affect whether voters observe evidential circumstances and how they interpret them.

What exactly is part of the circumstances (evidential or not)?
–> the **common causes/factors** of votes (in a Bayesian network)
(–> common = common to at least two voters)

The fixed-problem CJT (1)

Some authors or arguments defending independence rely (implicitly or explicitly) on a fixed decision problem
–> i.e. fixed correct alternative, fixed circumstances.

Ingredients of the "fixed-problem CJT":
Fixed correct alternative x in $\{0, 1\}$
Votes: random variables $V_1, V_2, ...$ in $\{0, 1\}$
A probability function Pr (on the underlying probability space)
- represents objective uncertainty *given* a fixed problem (as described by the correct alternative x fixed circumstances)

The fixed-problem CJT (2)

The probability function Pr:

- interpretable as $Pr = P(.|PROBLEM = problem)$, where
- P captures objective uncertainty **also about the problem**,
- $PROBLEM$ is a (highly multi-dimensional) random variable, $problem$ a realisation
- say $PROBLEM = (X, C)$, where C are random circumstances and X a random correct alternative

The fixed-problem CJT (3)

Independence (Ind). The votes $V_1, V_2, ...$ of individuals 1, 2, ... are independent.

Individual i's *competence (on the given problem)* is $p_i := Pr(V_i = x)$, the probability that vote V_i is correct

Competence on the problem (Com). Competence on the problem $p_i = Pr(V_i = x)$ exceeds 1/2 and is the same across individuals i.

Competence-on-average on the problem ($\overline{\text{Com}}$). Average competence on the problem $\bar{p} := \lim_{n\to\infty}(p_1 + ... + p_n)/n$ (exists and) exceeds 1/2.

The fixed-problem CJT. If (Ind) and ($\overline{\text{Com}}$) hold, the probability of a correct majority outcome, $Pr(\#\{i \leq n : V_i = x\} > n/2)$, tends to one as the group size n tends to infinity.

The fixed-problem CJT (4)

Proof. Assume (Ind) and ($\overline{\text{Com}}$).
Suppose $x = 1$ (the proof is analogous if $x = 0$).
The (centered) random variables $V_i - p_i$, $i = 1, 2, \ldots$,
- have zero expectation;
- are independent

So, by the law of large numbers,

$$Pr(\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n}(V_i - p_i) \to 0) = 1.$$

Using that $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} p_i \to \bar{p}$, it follows that

$$Pr(\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} V_i \to \bar{p}) = 1.$$

So (as $\bar{p} > \frac{1}{2}$ & as convergence with prob. $1 \Rightarrow$ stoch. conver.)

$$Pr(\frac{1}{n}\sum_{i=1}^{n} V_i \ > \ 1/2) \to 1,$$

i.e. $Pr\,(\#\{i \leq n : V_i = 1\} > n/2) \ \to \ 1.$ ■

The fixed-problem CJT (5)

Are the premises of the fixed-problem CJT justified?

- **Independence** premise can be defended

  –> on the basis of the Parental Markov condition (or Reichenbach's common-cause principle)

  –> provided the (fixed) circumstances are defined comprehensively, i.e. contain all common causes

- **Competence** can (usually) not be *known to hold*:

  –> knowing whether ($\overline{\text{Com}}$) holds *for this specific problem* might be even harder than knowing the true state x in the first stage.

  –> Reason: to know whether ($\overline{\text{Com}}$) holds, one would have to know whether the specific problem involves misleading evidence,

  –> which one can hardly know without knowing the true state x.

The variable-problem CJT (1)

Now think of the decision problem (including the circumstances: room temperature, body of evidence) as randomly generated

Formally:
Random state X in $\{0, 1\}$ (correct alternative)
Random votes $V_1, V_2, ...$ in $\{0, 1\}$
Probability measure P (not Pr!)
–> represents objective uncertainty under a random process generating people's votes *and* their decision problem.
–> where $P(X = 0)$ is strictly between 0 and 1 (to make the state genuinely random)

The variable-problem CJT (2)

Independence (IND). The votes $V_1, V_2, \ldots$ of individuals 1, 2, ... are independent conditional on $X = 0$, and also independent conditional on $X = 1$.

Individual i's *competence given that alternative x is correct*:
$p_i^x := P(V_i = x | X = x)$
–> the probability of voting for x given that x is correct.
Individual i's *(unconditional) competence*:
$p_i = P(V_i = X)$

Competence (COM). For each alternative $x \in \{0, 1\}$, conditional competence $p_i^x = P(V_i = x | X = x)$ exceeds 1/2 and is the same across individuals i.

The variable-problem CJT (3)

Competence-on-average ($\overline{\text{COM}}$) For each alternative $x \in \{0, 1\}$, average conditional competence $\bar{p}^x := \lim_{n\to\infty}(p_1^x + ... + p_n^x)/n$ (exists and) exceeds $1/2$.

The variable-problem CJT. If (IND) and ($\overline{\text{COM}}$) hold, the probability of a correct majority outcome, $P(\#\{i \leq n : V_i = X\} > n/2)$, and also for each alternative $x \in \{0, 1\}$ the conditional probability of a correct majority outcome, $P(\#\{i \leq n : V_i = x\} > n/2 | X = x)$, tend to one as the group size n tends to infinity.

The variable-problem CJT (4)

Are the premises of the variable-problem CJT justified?

- **Competence** is now (usually) justified

  –> provided the (random) problem-generating process picks the problem from a natural reference class of problems

  –> e.g. from the class of all convict-acquit problems

  –> or the class of all do-operation-or-not problems

  –> but not from the class of all misleading problems!

- **Independence** usually fails to hold

  –> because common causes of the votes aren't conditionalised upon

  –> usually, these common causes introduce **positive** correlation

  –> e.g. mistake by voters 1 increase probability of misleading evidence, which increases probability of mistake by voter 2

A social planer's belief in the majority judgment (1)

Now let probability be
- not objective (as before)
- but subjective, held by a social planner

Such a probability measure behaves more like the measure P in the variable-problem CJT
–> because the planner won't **know** the problem in all detail
–> he doesn't know which state is correct, and what the exact circumstances are

A social planer's belief in the majority judgment (2)

So:

independence is likely to fail

But the **competence** assumption is likely to hold

–> but "competence" isn't the right term, because p_i^x doesn't represent i's ability but the planner's belief in i's ability

–> the planner may take a genious for an idiot

Ladha (1993) weakens independence to **voter exchangeability**

–> in de Finetti's sense

–> a plausible relaxation, because the planner can have symmetric knowledge about individuals (despite their objective differences)