

# Tenability Pursuit

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January 13, 2012

## Abstract

Recently Hájek prophesied that “as fans of the *Friday the 13th* series know, you should never be surprised if Jason suddenly reappears in a new guise – and the same is true for Stalnaker’s Thesis” (Hájek, 2011, p. 14). So what better day to pick for a plea for the thesis than today? Obviously, a new guise is also needed for the thesis; it cannot live on in the way as it is understood in the triviality results. In this article I investigate the possibilities for the thesis if one follows (van Fraassen, 1976) in assuming that the meaning of a conditional may be context-sensitive. Several triviality proofs among which two new ones will be discussed, thereby investigating the necessity for, and identity of the context-sensitivity. These findings are then compared to the tenability results by van Fraassen, and I will argue that his logic *CE* does not yet take into account the context-sensitivity fully and correctly. Finally, also some direct arguments, not based on the thesis, in favor of context-sensitivity are presented.

## 1 Introduction

There is a growing consensus that there isn’t much hope for the future of Stalnaker’s thesis or, as I prefer to call it, the thesis.<sup>1</sup> This consensus is mainly due to a sea of so-called triviality results: (Lewis, 1976; Stalnaker, 1976; Rehder, 1982; Lewis, 1986; Hájek, 1989; Hájek & Hall, 1994; Hall, 1994; Milne, 2003; Douven & Dietz, 2011; Hájek, 2011). In this paper I will even expand this sea with two new results in section 4. However, as it is well known, van Fraassen found a small island in this sea that seems well protected against possible floods. In this paper I investigate the possibility of more islands by first investigating how they can be protected against floods in sections 3 and 4. Then in section 5 I will sketch the contours of such islands. In section 6 I will compare my results with those by van Fraassen with a focus on the recent attack against van Fraassen found in (Douven & Dietz, 2011). Finally, in section 7 I will defend the most criticized assumption made by van Fraassen and me, namely, the context-sensitivity of the meaning of the conditional.

Before I get to these results it is best to set a background which will be helpful in the main investigations. The big question I am interested in is the one that burdens many: “what is a conditional?” For the logical connectives ‘and’, ‘or’ and ‘not’ the rules for manipulating their mathematical counterparts ( $\wedge$ ,  $\vee$  and  $\neg$ ) in logic are widely accepted. That is, the rules are compatible with the meaning of the connectives in natural language. For example, a sentence of the form  $A \vee (B \wedge C)$  is thought to be true if and only if the sentence  $(A \vee B) \wedge (A \vee C)$  is true. But the conditional is a notorious exception. The classical way of capturing the use of “if... then ...” in the form of the material implication (denoted by  $\supset$  in this text) is considered to be unsatisfactory by many philosophers. For example, according to the material implication the sentence “If John is in Paris, then he is in London or, if John is in London, then he is in Paris.” is a tautology. But in many cases (if not all) it seems to me that neither of the disjuncts are

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<sup>1</sup>The thesis is often attributed to Stalnaker specifically due to his advocacy of it in (Stalnaker, 1970). The reason for using the term thesis instead is fourfold: 1) Stalnaker himself no longer supports it (Stalnaker, 1976). 2) It is also sometimes attributed to Adams (Adams, 1975) or Ramsey (Ramsey, 1931). 3) The term ‘thesis’ is also used often (Lewis, 1976; van Fraassen, 1976). 4) It is shorter.

true.<sup>2</sup> So then the main question arises: if the material implication does not capture the meaning of conditionals, what logical construction does?

The question may perhaps be evaded if one denies that a sentence in the form of a conditional can express a proposition (i.e. there is a fact of the matter as to which the sentence is either true or false). But consider as an example the sentence “If I am a bachelor, then I am not married.” What does this express if not a proposition that is true and even a tautology? But also less trivial conditionals appear to me to be propositions. For example the sentence “If I still have some cans of beans, then I won’t starve.” may be true or false. Conditions for it to be true may include for example “I have a can opener.” or “I am not allergic to beans.”

So now the stage is set for the investigation and also some results are already in place. To answer the big question is to introduce a formalization  $A \rightarrow B$  of the proposition “If  $A$ , then  $B$ ” for certain pairs of propositions  $A$  and  $B$ . To be clear, this is something that will not be achieved in this paper, but the possibilities will be narrowed down. Now I do not dare to claim that the construction “If  $A$ , then  $B$ ” always leads to a proposition and thus I do not assume that such a sentence can always be represented by  $A \rightarrow B$ . But there are at least some cases for which it must work. Furthermore, from the ‘beans’-example it may be clear that I don’t believe that the truth value of  $A \rightarrow B$  is completely determined by the truth values of  $A$  and  $B$ ; it may well depend on the truth values of many other propositions. That is, it will depend on the *context* in which  $A \rightarrow B$  is stated.

This is the point where it becomes convenient to follow Stalnaker and Lewis and adopt a possible worlds approach. However, I will refrain from assuming any form of nearness relations between possible worlds. A context now coincides with a set of possible worlds and so, for example, there may be contexts in which  $A$  is false,  $B$  is true and  $A \rightarrow B$  is true, but also contexts in which  $A$  is false,  $B$  is true and  $A \rightarrow B$  is false. A priori, the full scope of possibilities can be depicted using a Karnaugh map

$$A \rightarrow B \left[ \begin{array}{|c|c|c|c|} \hline & \overbrace{\hspace{1.5cm}}^A & & \\ \hline & & & \\ \hline & & & \\ \hline & \underbrace{\hspace{1.5cm}}_B & & \\ \hline \end{array} \right. \quad (1)$$

where the total rectangle represents the set of all possible worlds, the first row the set of all possible worlds in which  $A \rightarrow B$  is true, the second row the set of all possible worlds in which  $A \rightarrow B$  is false, etc. Presumptions about what rules should hold for the conditional may rule out some of these possibilities. For example, according to modus ponens (MP) it is not possible that  $A$  is true and  $A \rightarrow B$  is true and  $B$  is false, thereby eliminating the upper left corner from all possibilities.

But obviously the task isn’t to eliminate possibilities for that would imply that the truth value of  $A \rightarrow B$  *does* depend on the truth values of  $A$  and  $B$  alone. On the other hand, leaving all options open doesn’t provide much insight in the meaning of conditionals. No, the tricky part is to get the hang of the role of the context without going into the mess of fully specifying all possible worlds (i.e. adding all propositions that may influence the truth value of  $A \rightarrow B$ ) and this is where the thesis comes to the stage, as I will explain in the next section.

## 2 Probabilities

A clever way of letting the context play a role without specifying it, is by looking at probabilities. Indeed, the probabilities one assigns to the propositions  $A$ ,  $B$  and Boolean combinations thereof is influenced by the context. That is, if the meaning of a proposition alters from context to context, then this will be reflected in the change in the probability assigned to that proposition upon changing the context. The hope is that studying the change in a probability function due to a change in context is enough to capture the change in the meaning of a proposition due to the same change in context.

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<sup>2</sup>An extensive discussion of the pros and cons of the material conditional may be found for example in (Bennett, 2003).

A difficulty is that usually probability functions do not yet specify the probability one should assign to  $A \rightarrow B$  or Boolean combinations of this proposition with  $A$  and/or  $B$ . A priori one has the following possibilities

$$A \rightarrow B \left[ \begin{array}{|c|c|c|c|} \hline & \overbrace{\hspace{4cm}}^A & & \\ \hline q_1 p_1 & q_2 p_2 & q_3 p_3 & q_4 p_4 \\ \hline \bar{q}_1 p_1 & \bar{q}_2 p_2 & \bar{q}_3 p_3 & \bar{q}_4 p_4 \\ \hline & \underbrace{\hspace{4cm}}_B & & \\ \hline \end{array} \right] \quad (2)$$

with  $0 \leq p_i \leq 1$ ,  $0 \leq q_i \leq 1$ ,  $q_i + \bar{q}_i = 1$  and  $p_1 + p_2 + p_3 + p_4 = 1$ . Naturally, when a probability function is specified, only the numbers  $p_1, \dots, p_4$  are specified and the values one should take for  $q_1, \dots, q_4$  are still open for discussion.

As far as I know not many attempts have been made to investigate possible values for the  $q_i$ 's, but at least one attempt has become persistently popular in the literature:

“If two people are arguing ‘If  $p$ , will  $q$ ?’ and are both in doubt as to  $p$ , they are adding  $p$  hypothetically to their stock of knowledge and arguing on that basis about  $q$  [...] We can say they are fixing their degrees of belief in  $q$  given  $p$ ” (Ramsey, 1931, p. 247)

In the present formulation this comes down to the demand

$$\text{If } 0 < P[A] < 1, \text{ then } P[A \rightarrow B] = P[B|A]. \quad (3)$$

Or, as a restriction on the  $q_i$ 's:

$$q_1 p_1 + q_2 p_2 + q_3 p_3 + q_4 p_4 = \frac{p_2}{p_1 + p_2}, \text{ whenever } 0 < p_1 + p_2 < 1. \quad (4)$$

In line with (Lewis, 1976) I shall refer to this demand as the thesis. Ramsey's proposal has much appeal. First of all, it has the virtue of appearing true. The example of van Fraassen is very compelling indeed:

“What is the probability that I throw a six if I throw an even number, if not the probability that: if I throw an even number, it will be a six?” (van Fraassen, 1976, p. 273)

Secondly, it appears as a very weak assumption mathematically. But as natural as the thesis sounds, as mentioned earlier, there are many results that indicate that it is untenable. In this text I will add some to these results, but also propose some objections against them, and some ideas for how to continue in the search for a formalization of conditionals. Indeed, I take the so-called triviality results as an opportunity to revise assumptions on conditionals, and not as a hint that conditionals have no truth values after all. In that sense I side myself with Lewis' view that rejecting truth values “burdens us with too much work still to be done, and wastes too much that has been done already.” (Lewis, 1976, p. 305)

### 3 Triviality Results

The first result I will discuss is due to Hájek. It establishes in a simple way that, if the thesis is presumed to be true, then the conditional cannot be truth-functional. This follows from the lottery example:

“Consider a 3-ticket lottery, and let  $L_i =$  ticket  $i$  wins,  $i = 1, 2, 3$ . Model it with the obvious 3-world probability space, with  $P[L_i] = 1/3$  for all  $i$ . Then the conditional probability  $P[L_1|L_1 \vee L_2]$  is obviously  $1/2$ . But all unconditional probabilities are a multiple of  $1/3$ , so in particular the unconditional probability  $P[(L_1 \vee L_2) \rightarrow L_1]$  is a multiple of  $1/3$ . And  $1/2$  is not a multiple of  $1/3$ .” (Hájek, 2011, p. 4)

Indeed, if the conditional  $L_1 \vee L_2 \rightarrow L_1$  were to be truth-functional, it would coincide with one of the propositions  $\perp$ ,  $L_1$ ,  $L_2$ ,  $L_3$ ,  $L_2 \vee L_3$ ,  $L_1 \vee L_3$ ,  $L_1 \vee L_2$  or  $\top$  and its probability would be a multiple of  $\frac{1}{3}$ . The thesis thus forces one to extend the set of all propositions. This is not so surprising at first. However, Hájek also showed in (Hájek, 1989) that if one wishes that the thesis holds for a set of propositions that is closed under Boolean operations and making conditionals, this set has to be infinite.

This result was presented as an argument against the thesis. But in my opinion, in this light it is a very poor argument for it only works as such if one firmly holds on to two assumptions that are both questionable. The first is that the set of all propositions should be finite. Of course, in practice one may be able to only formulate a finite number of propositions, but this does not prohibit one to think that those propositions that can be formulated just form a part of the infinite set of possible propositions; those that can be formulated potentially. In physics this idea is standard; for a fixed particle, say an electron, and any point in space  $x$ , one can formulate the proposition “The electron is positioned at the point  $x$ ”. Since there are infinitely many points in space to pick from, an infinite number of proposition can be formulated potentially. So the idea of an infinite number of propositions is fully acceptable, if only as an idealization.

Now, the strict finitist may reject this idealization, but he/she will most likely also reject the second assumption needed for Hájek’s argument. Namely, that the set of propositions is closed under forming conditionals. That is, the assumption that it makes sense to formulate sentences of *arbitrary length* using an arbitrary number of “if ..., then ...” constructions. This is a strong assumption given that there are lengthy discussions about whether or not it even makes sense to have two or three of such constructs in one sentence. I think that those who find it unproblematic to form sentences with more constructs will most likely also find it unproblematic to accept the possibility of an infinite number of propositions. Thus accepting the second assumption gives reason for rejecting the first.

It may be mentioned that, strictly speaking, the two assumptions aren’t contradictory. The material implication is a good example of a conditional that satisfies both. But this doesn’t give reason to assume that therefore the two assumptions are obvious assumptions that should be satisfied by any conditional. In fact, I find it rather special that the material implication satisfies both and this may even be seen as an explanation of why it only crudely captures the notion of a conditional.

The most famous result that argues against the thesis comes from Lewis’ 1976 paper.

**Theorem 1.** *If  $L$  is a set of propositions closed under the Boolean operations  $\vee$ ,  $\wedge$  and  $\neg$  and if  $\mathcal{P}$  is a set of probability functions on  $L$  that is closed under conditioning (i.e. if  $P \in \mathcal{P}$  and  $A \in L$ , then  $P[\cdot|A] \in \mathcal{P}$ ) and satisfy the thesis, then for any pair of propositions  $A, B \in L$  with  $A \rightarrow B \in L$  and every  $P \in \mathcal{P}$  with  $0 < P[B] < 1$  and  $0 < P[A] < 1$*

$$P[B|A] = P[B]. \quad (5)$$

*Proof.* The proof consists of three steps. First apply the thesis to the conditional  $A \rightarrow B$ :

$$P[B|A] = P[A \rightarrow B]. \quad (6)$$

Now apply the law of total probability (this is justified because  $L$  is assumed to be closed under Boolean operations):

$$P[A \rightarrow B] = P[A \rightarrow B|B]P[B] + P[A \rightarrow B|\neg B]P[\neg B]. \quad (7)$$

Finally, apply the thesis to the conditional probabilities:

$$P[A \rightarrow B|B]P[B] + P[A \rightarrow B|\neg B]P[\neg B] = P[B|A \wedge B]P[B] + P[B|A \wedge \neg B]P[\neg B] = P[B]. \quad (8)$$

□

Now anyone who wishes to uphold the thesis has to find objections to the second or third step in the proof and hence has to object against the assumptions used to justify these steps. A

popular response is to deny that conditionals express propositions and thus have no truth values. The thesis then no longer expresses a degree of belief for the conditional being true, but rather a degree of assertability or acceptability. In this form the thesis lives on under the name of Adams's thesis, after Adams's proposal in (Adams, 1975).

Others may say that conditionals do have truth values, but deny that Boolean combinations involving conditionals express propositions. This would lead to a rejection of the law of total probability for conditionals. Personally, I think that if  $A \rightarrow B$  expresses a proposition, then so does any Boolean combination of it with other propositions. And that is actually all that is needed to justify the law of total probability.<sup>3</sup>

Objections against the third step go way back to (van Fraassen, 1976). Although the third step hinges on the assumption that  $\mathcal{P}$  is closed under conditionalization, this is not the assumption attacked by van Fraassen. Instead, he points to an auxiliary assumption: 'the fixed interpretation of  $\rightarrow$ '. This assumption was later termed 'metaphysical realism' by (Stalnaker, 1976). However, I prefer a less laden term and speak instead of the denial of the context-sensitivity of the conditional. So let me explain how it enters in Lewis' result. When the thesis is applied again to  $P[A \rightarrow B|B]$  two contexts are being mixed so to speak. The argument in this probability ( $A \rightarrow B$ ) is specified with respect to the context in which the unconditional probability function  $P$  is specified. As the context changes to one in which  $P[.|B]$  is specified, also the meaning of the symbol  $\rightarrow$  changes. That is, the way in which its truth is evaluated changes. Upon learning  $B$ , not only the degrees of belief in the propositions  $A$ ,  $B$  and  $A \wedge B$  change, but also those in all other propositions that may influence the truth value of  $A \rightarrow B$ , and this is what seems to be neglected in Lewis' proof. The picture with the  $q_i$ 's shows how strong this auxiliary assumption actually is. After conditioning on  $B$  the table in (2) becomes

$$A \rightarrow B \left[ \begin{array}{c|cc|c} \overbrace{\hspace{1.5cm}}^A & 0 & q_2 \frac{p_2}{p_2+p_3} & q_3 \frac{p_3}{p_2+p_3} & 0 \\ \hline & 0 & \overbrace{q_2 \frac{p_2}{p_2+p_3}}^B & \overbrace{q_3 \frac{p_3}{p_2+p_3}}^B & 0 \end{array} \right. \quad (9)$$

Now suppose the  $q_i$ 's are fully determined by the probability function, i.e. there are general rules  $f_i$  such that for any probability function  $q_i = f_i(p_1, p_2, p_3, p_4)$ . The assumption then boils down to the restriction

$$f_2(p_1, p_2, p_3, p_4) = f_2\left(0, \frac{p_2}{p_2+p_3}, \frac{p_3}{p_2+p_3}, 0\right) \text{ and } f_3(p_1, p_2, p_3, p_4) = f_3\left(0, \frac{p_2}{p_2+p_3}, \frac{p_3}{p_2+p_3}, 0\right). \quad (10)$$

When comparing with the formulation of the thesis in (4) it does indeed seem strange to demand that the values for  $q_2$  and  $q_3$  in (9) should be equal to those in (2); if the  $p_i$ 's change under conditionalization, then why not the  $q_i$ 's?

Whereas Lewis' triviality proof thus relies on a certain restriction on the values for the  $q_i$ 's, another triviality proof, (Hájek, 1994), relies on quite the opposite. In his 'Triviality on the cheap?' Hájek considers so-called perturbations of a probability function  $P$  for which the thesis does hold. A perturbation may be thought of as a slight adjustment in ones beliefs. Obviously, specific perturbations are considered, namely, those where only some of the  $q_i$ 's are slightly altered, but the values of the  $p_i$ 's and the other  $q_i$ 's remain constant. Of course this is enough to violate the thesis. This becomes clear if one looks at its formulation in (4). In other words, the thesis is violated because Hájek assumes that one can rationally set ones degrees of belief in a way such that the thesis is violated. This hinges on the cheap indeed.

It is possible to make the argument sound less cheap. The following quote does indeed sound compelling:

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<sup>3</sup>That, and the assumption that probability functions obey some standard rules and the Boolean operations are correct.

“A perturbation of a probability function can differ ever so slightly from that function [...] It is hard to see how one of these functions can be a belief function, and the other not.” (Hájek, 1994, p. 125)

But it is not hard to see how perturbations can allow one to go from a belief function to a non-belief function for other logical connectives. Consider for example probability functions for disjunctions:

$$A \vee B \left[ \begin{array}{|c|c|c|c|} \hline & \overbrace{\hspace{2cm}}^A & & \\ \hline q_1 p_1 & q_2 p_2 & q_3 p_3 & q_4 p_4 \\ \hline \overline{q}_1 p_1 & \overline{q}_2 p_2 & \overline{q}_3 p_3 & \overline{q}_4 p_4 \\ \hline & \underbrace{\hspace{2cm}}_B & & \\ \hline \end{array} \right] \quad (11)$$

This represents a belief function iff  $q_1 = q_2 = q_3 = \overline{q}_4 = 1$ , and even the slightest variation in these values would result in nonsense. The difference is of course that the values for the  $q_i$ 's for disjunction are extremal. This is because disjunction is truth functional. But it is a huge leap to go from denying truth functionality for the conditional to allowing *all* possible values for the  $q_i$ 's *independent* of what one believes about  $A$  and  $B$ . If this were allowed, then a similar argument based on perturbations may also be used to argue against MP or any other widely accepted rule for conditionals.

Hájek does consider the possible reply that only probability functions that obey the thesis may be admissible for a rational agent. This is indeed the reply I would give when defending the thesis, but Hájek rejects it on the ground that it lacks plausibility. This is quite a strange rejection in my opinion, for the main reason for assuming the thesis *is* that it sounds plausible. What Hájek seems to be asking for is another motivation for the thesis, perhaps in the form of a Dutch book argument. But this seems too much to ask, for any deeper motivation for the thesis can only rely on a deeper understanding of what the conditional is. Furthermore, the plausibility argument seems to have no connection whatsoever with the perturbation argument. Obviously, if one thinks the thesis doesn't sound plausible, one should reject it. But for those who do think it sounds plausible there seems to be nothing in (Hájek, 1994) to make them change their minds.

Although I agree with van Fraassen's objection, it is not a very fruitful one to obtain new information about what a conditional is, and *how* its meaning depends on the context. It only follows that Lewis' result has no direct consequences for those who believe that the meaning of a conditional is context-sensitive. But after accepting context-sensitivity, it seems to me there is still a wide range of possibilities. Van Fraassen's tenability results provide one way to go, but for me it is not a satisfactory one. In section 6 I will return to this issue.

Besides the objection against the auxiliary assumption, I also have a direct objection against the assumption that  $\mathcal{P}$  should be closed under conditionalization. In fact, the assumption itself is in conflict with the thesis as I presented it here which has the clause  $0 < P[A] < 1$ . Obviously, if  $\mathcal{P}$  is closed under conditionalization, then not every element of it can satisfy the thesis, because it is also closed under conditioning on  $A$  and  $\neg A$  and it is clear that the thesis doesn't hold for  $P[A \rightarrow B | \neg A]$ . The clause is not just a technicality. Sure it is necessary that  $P[A] > 0$  in order for  $P[B|A]$  to even be defined, but conditionals also become conspicuous if the antecedent is true or false with certainty. And the same holds for the consequent, the whole point of the use of conditionals is based on uncertainty of the antecedent and the consequent. Or in the words of Edgington:

“Our interest in conditionals centers on the case where we're not sure whether  $A$ , not sure whether  $B$ , but the supposition that  $A$  has some bearing under whether  $B$ .” (Edgington, 1995, p. 268)

So I would say that the conditional  $A \rightarrow B$  is poorly defined if one is certain about either  $B$  or  $\neg B$ , thereby rejecting the third step in Lewis' proof and the assumption that  $\mathcal{P}$  can be closed under conditionalization. More specifically, I find it troublesome that the faith of the thesis is being settled relying on such difficult situations as appearing in the proof of Lewis.

Thus far my objections can be summed up as follows.

- 1) It is reckless to apply the thesis to a conditional in a conditional probability function whereas the conditional itself was originally posed within a context where an unconditional probability function holds.
- 2) The triviality result seems to rely on rather conspicuous situations, i.e. conditioning on the consequent or the negation of the consequent.

I admit that these objections are still a bit vague and they need to be explained better. I will do this as I proceed with discussing more triviality results.

## 4 The Plot Thickens

There is a quick fix for the first objection although I think it is not known as such.<sup>4</sup> It involves an adjustment in the proof of Lewis, avoiding the assumption that  $\mathcal{P}$  should be closed under conditionalization.<sup>5</sup> More specifically, the proof of this result requires only that the thesis holds for a single probability function and thus requires no change in context due to a change in the probability function in that sense. Of course this fix comes with a prize tag. As an additional constraint for the conditional, the law of Import-Export (IE) is assumed, namely that

$$A \rightarrow (B \rightarrow C) = (A \wedge B) \rightarrow C. \quad (12)$$

**Theorem 2.** *Let  $L$  be a set of propositions closed under the Boolean operations  $\vee$ ,  $\wedge$  and  $\neg$  and let  $\rightarrow$  be a conditional that satisfies (IE) whenever both sides of (12) are defined in  $L$ . If  $P$  is a probability function on  $L$  that satisfies the thesis, then for any pair  $A, B \in L$  for which  $A \rightarrow B$ ,  $B \rightarrow (A \rightarrow B)$ ,  $\neg B \rightarrow (A \rightarrow B)$ ,  $(A \wedge B) \rightarrow B$  and  $(A \wedge \neg B) \rightarrow B$  are all in  $L$  one has*

$$P[B|A] = P[B]. \quad (13)$$

*Proof.* The first two steps are identical to the ones in the proof of theorem 1, but the third step now involves some extra steps. First apply the thesis to the conditionals  $B \rightarrow (A \rightarrow B)$  and  $\neg B \rightarrow (A \rightarrow B)$ :

$$P[A \rightarrow B|B]P[B] + P[A \rightarrow B|\neg B]P[\neg B] = P[B \rightarrow (A \rightarrow B)]P[B] + P[\neg B \rightarrow (A \rightarrow B)]P[\neg B]. \quad (14)$$

Then IE states

$$\begin{aligned} &P[B \rightarrow (A \rightarrow B)]P[B] + P[\neg B \rightarrow (A \rightarrow B)]P[\neg B] \\ &= P[(B \wedge A) \rightarrow B]P[B] + P[(\neg B \wedge A) \rightarrow B]P[\neg B]. \end{aligned} \quad (15)$$

Applying the thesis to the conditionals  $(B \wedge A) \rightarrow B$  and  $(\neg B \wedge A) \rightarrow B$  then gives the desired result.  $\square$

It is a natural response for the defender of the thesis to put the blame on IE. However, this assumption has a same kind of appeal as the thesis; in conceivable situations it just sounds right. So one would need an argument that shows that the one sounds more right than the other. Now in many cases IE sounds right because  $(A \wedge B) \rightarrow C$  presents itself as a good explanation of  $A \rightarrow (B \rightarrow C)$ . In other words, the appeal of IE is a bit suspicious. This is not to say that  $A \rightarrow (B \rightarrow C)$  may well have a different meaning and that we are just not bright enough to distinct it from  $(A \wedge B) \rightarrow C$ , but it does indicate that our intuitions about sentences of the form  $A \rightarrow (B \rightarrow C)$  are very poor. However, the thesis is built upon intuition alone. But if our intuition of nested conditionals is known to be poor, what reason is there left to assume that the thesis will also hold for these nested conditionals?

<sup>4</sup>I will postpone discussion of the second objection until section 5.

<sup>5</sup>A premature version of this spin-off of Lewis' proof may be found in (Blackburn, 1994). See also (Douven, 2012) for a clear presentation.

Let me elaborate on this point a bit. One part of what makes the equation  $P[A \rightarrow B] = P[B|A]$  so appealing, is that we have a strong feeling for what  $P[B|A]$  expresses. But in the case of  $P[B \rightarrow C|A]$  this is no longer the case because  $B \rightarrow C$  expresses a proposition whose meaning is understood in the context before conditioning on  $A$ . In other words, if one accepts the context-dependence of conditionals, the interpretation of  $P[B \rightarrow C|A]$  is yet another mystery to be solved. One possible way to go is to make a leap of faith and assume that this is precisely the number that expresses  $P[A \rightarrow (B \rightarrow C)]$  (thus assuming the thesis also for nested conditionals). And the other way is to state that it is nothing but the quotient  $P[(B \rightarrow C) \wedge A]/P[A]$  and that it has in itself no significant meaning. This last option thus assumes that the standard rule for conditioning in probability theory is not adequate for conditionals. That is, the equation

$$P[A \rightarrow (B \rightarrow C)] = P[B \rightarrow C|A] \quad (16)$$

will be rejected in general. This is the path I will follow. Note that if one assumes IE, one does have a natural extension for the thesis for (right-)nested conditionals:

$$P[A \rightarrow (B \rightarrow C)] = P[C|A \wedge B]. \quad (17)$$

There are no triviality results for a conditional that is context-dependent, satisfies the thesis for simple conditionals, and satisfies IE. This can in fact be seen from van Fraassen's tenability result. Although his minimal logic  $CE$  (van Fraassen, 1976, p. 277) only involves simple conditionals, IE can be added without problems. Indeed, if one understands it simply as a definition for the right-nested conditional there are no consequences, as long as one denies (16). In other words, these rules simply seem to be too weak to lay any destructive constraints on the admissible probability functions. That is, for any choice of  $p_1, p_2, p_3, p_4$   $q_i$ 's can be found such that the assumptions are all satisfied. It may be noted that van Fraassen also looked into nested conditionals, but only for Stalnaker semantics. Or, to be more precise, a specific extension of  $CE$  inspired by Stalnaker semantics. Either way, for such semantics, IE is most likely to be rejected.

The above set of assumptions (the thesis for simple conditionals together with IE) seem to lead to a nice tenability result, but unfortunately adding only the smallest of constraints for the right-nested conditional causes the system to collapse. This is demonstrated by the the following two theorems that I will discuss which are, as far as I know, new to the discussion. The first one concerns the consequences for adding the constraint that  $A \wedge B$  implies  $A \rightarrow B$  whenever the last one is defined.

**Theorem 3.** *Suppose  $\rightarrow$  is a conditional that satisfies IE and furthermore that whenever  $A$  and  $B$  are true, then also  $A \rightarrow B$  is true. If  $P$  is a probability function on  $L$  that satisfies the thesis for simple conditionals, then for any pair  $A, B \in L$  with  $A \rightarrow B \in L$ ,  $(\neg B \wedge A) \rightarrow B \in L$  and  $(\neg A \vee \neg B) \rightarrow (A \rightarrow B) \in L$  one has*

$$P[B|A] = P[A \wedge B] \text{ and } P[\neg A] = 0. \quad (18)$$

*Proof.* The full range of possible probability assignments for Boolean combinations of  $A, B, A \rightarrow B$  and  $(\neg A \vee \neg B) \rightarrow (A \rightarrow B)$  is given by

$$A \rightarrow B \left[ \begin{array}{c} \overbrace{\hspace{10em}}^A \\ \begin{array}{|c|c|c|c|} \hline \overline{r_1}q_1p_1 & \overline{r_2}q_2p_2 & \overline{r_3}q_3p_3 & \overline{r_4}q_4p_4 \\ \hline r_1q_1p_1 & r_2q_2p_2 & r_3q_3p_3 & r_4q_4p_4 \\ \hline r_1\overline{q_1}p_1 & r_2\overline{q_2}p_2 & r_3\overline{q_3}p_3 & r_4\overline{q_4}p_4 \\ \hline \overline{r_1}\overline{q_1}p_1 & \overline{r_2}\overline{q_2}p_2 & \overline{r_3}\overline{q_3}p_3 & \overline{r_4}\overline{q_4}p_4 \\ \hline \end{array} \\ \underbrace{\hspace{10em}}_B \end{array} \right] (\neg A \vee \neg B) \rightarrow (A \rightarrow B) \quad (19)$$

where the  $p_i$ 's and  $q_i$ 's are as before, and  $0 \leq r_i \leq 1$ ,  $r_i + \overline{r_i} = 1$ .

According to IE  $(\neg A \vee \neg B) \rightarrow (A \rightarrow B) = (A \wedge \neg B) \rightarrow B$ . Applying the thesis to this simple conditional amounts to  $P[(\neg A \vee \neg B) \rightarrow (A \rightarrow B)] = 0$ . So  $r_1 = r_2 = r_3 = r_4 = 0$ .

Furthermore, from the assumption it follows that  $A \wedge B \wedge \neg(A \rightarrow B)$  is a logical impossibility, and so is  $(\neg A \vee \neg B) \wedge (A \rightarrow B) \wedge \neg((\neg A \vee \neg B) \rightarrow (A \rightarrow B))$ . Incorporating these considerations amounts to

$$A \rightarrow B \left[ \begin{array}{c} \overbrace{\hspace{1.5cm}}^A \\ \begin{array}{|c|c|c|c|} \hline \perp & p_2 & \perp & \perp \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & \perp & 0 & 0 \\ \hline p_1 & \perp & p_3 & p_4 \\ \hline \end{array} \\ \underbrace{\hspace{1.5cm}}_B \end{array} \right] (\neg A \vee \neg B) \rightarrow (A \rightarrow B) \quad (20)$$

where it has been used that an additional consequence is that  $q_1 = \overline{q_2} = \overline{q_3} = \overline{q_4} = 1$ .

It now follows immediately from the thesis that

$$P[A \wedge B] = p_2 = P[A \rightarrow B] = P[B|A] = \frac{p_2}{p_1 + p_2}. \quad (21)$$

Thus  $p_1 + p_2 = 1$  and  $P[\neg A] = p_3 + p_4 = 0$ .  $\square$

It may be clear that this result not just amounts to triviality, but also to contradiction. For if the same assumptions are also to hold for the conditional  $\neg A \rightarrow B$  the total analysis amounts to  $P[A] = 0$  and hence  $1 = P[\top] = P[A \vee \neg A] = 0$ .

A similar result is obtained if one assumes MP instead of the assumption that  $A$  and  $B$  together imply  $A \rightarrow B$ . That the conjunction of MP with IE leads to peculiar results was already demonstrated in (Gibbard, 1981, p.234-235). In this text Gibbard showed that, if one also assumes that  $A \leq B$  (i.e. whenever  $A$  is true,  $B$  is true) implies that  $A \rightarrow B$  is true, the conditional must be equal to the material implication. This third assumption can actually be seen as a consequence of the thesis, with the difference that from the thesis it follows that if  $A \leq B$ , then one must assign probability 1 to the conditional  $A \rightarrow B$ . It is constructive to see how this result is obtained explicitly within the present setting.

**Theorem 4.** *Suppose  $\rightarrow$  is a conditional that satisfies IE and MP. If  $P$  is a probability function on  $L$  that satisfies the thesis for simple conditionals, then for any pair  $A, B \in L$  with  $A \rightarrow B \in L$ ,  $(A \wedge B) \rightarrow B \in L$  and  $(A \supset B) \rightarrow (A \rightarrow B) \in L$  one has*

$$P[B|A] = P[A \supset B] \text{ and } P[\neg A] = 0. \quad (22)$$

*Proof.* The full range of possible probability assignments for Boolean combinations of  $A, B, A \rightarrow B$  and  $(A \supset B) \rightarrow (A \rightarrow B)$  is given by

$$A \rightarrow B \left[ \begin{array}{c} \overbrace{\hspace{1.5cm}}^A \\ \begin{array}{|c|c|c|c|} \hline \overline{r_1}q_1p_1 & \overline{r_2}q_2p_2 & \overline{r_3}q_3p_3 & \overline{r_4}q_4p_4 \\ \hline r_1q_1p_1 & r_2q_2p_2 & r_3q_3p_3 & r_4q_4p_4 \\ \hline r_1\overline{q_1}p_1 & r_2\overline{q_2}p_2 & r_3\overline{q_3}p_3 & r_4\overline{q_4}p_4 \\ \hline \overline{r_1}\overline{q_1}p_1 & \overline{r_2}\overline{q_2}p_2 & \overline{r_3}\overline{q_3}p_3 & \overline{r_4}\overline{q_4}p_4 \\ \hline \end{array} \\ \underbrace{\hspace{1.5cm}}_B \end{array} \right] (A \supset B) \rightarrow (A \rightarrow B) \quad (23)$$

According to IE  $(A \supset B) \rightarrow (A \rightarrow B) = (A \wedge B) \rightarrow B$ . Applying the thesis to this simple conditional amounts to  $P[(A \supset B) \rightarrow (A \rightarrow B)] = 1$ . So  $r_1 = r_2 = r_3 = r_4 = 1$ . Furthermore, from MP it follows that  $A \wedge (A \rightarrow B) \wedge \neg B$  is a logical impossibility, and so is  $(A \supset B) \wedge ((A \supset$

$B) \rightarrow (A \rightarrow B)) \wedge \neg(A \rightarrow B)$ . Incorporating these considerations amounts to

$$A \rightarrow B \left[ \begin{array}{c|c|c|c} \hline & \text{A} & & \\ \hline \perp & 0 & 0 & 0 \\ \hline \perp & p_2 & p_3 & p_4 \\ \hline p_1 & \perp & \perp & \perp \\ \hline 0 & 0 & 0 & 0 \\ \hline & \text{B} & & \\ \hline \end{array} \right] (A \supset B) \rightarrow (A \rightarrow B) \quad (24)$$

where it has been used that an additional consequence is that  $q_1 = \bar{q}_2 = \bar{q}_3 = \bar{q}_4 = 0$ .

It now follows immediately from the thesis that

$$1 - p_1 = p_2 + p_3 + p_4 = P[A \rightarrow B] = P[B|A] = \frac{p_2}{p_1 + p_2}. \quad (25)$$

Consequently

$$p_1 = p_1(p_1 + p_2) \quad (26)$$

and thus  $P[\neg A] = 1 - (p_1 + p_2) = 0$ .  $\square$

Again extending these results also to the conditional  $\neg A \rightarrow B$  leads to a contradiction.

## 5 Towards Tenability

Theorem 4 is what I consider the most compulsory result about conditionals. It shows that three strong intuitions about conditionals (MP, IE and the thesis for simple conditionals) are incompatible. But simply pointing out one of those as the guilty one seems too reckless to me, and a more careful analysis is required. It is time to take a closer look at my earlier objections 1) and 2). Let's start with the second.

What makes the discussion somewhat more confusing is that all of the theorems explicitly consider nested conditionals that are somewhat contrived. Sentences like  $B \rightarrow (A \rightarrow B)$  have a tendency to sound strange for many choices of  $A$  and  $B$ . As an example consider  $B$ ="I bring an umbrella" and  $A$ ="It doesn't rain". The sentence "If I bring an umbrella then, if it doesn't rain, I bring an umbrella." sounds weird. Using IE and the thesis one should assign probability one to this proposition. This can be understood as follows: "If I bring an umbrella, then it doesn't matter if it rains or not, I still have it with me." Similarly,  $B \rightarrow (A \rightarrow \neg B)$  is always false. But it is odd to formulate it as  $B \rightarrow (A \rightarrow B)$  or  $B \rightarrow (A \rightarrow \neg B)$ . Moreover, if I see the sentence spelled out as  $B \rightarrow (A \rightarrow \neg B)$ , I have a tendency to formulate it as: "If I bring an umbrella, then if it hadn't rained, I would not have brought an umbrella." In other words, the only reason for bringing an umbrella is if it would rain. Now I understand this is a way of cheating because in this last example ( $A \rightarrow \neg B$ ) is spelled out as a subjunctive conditional and, as it goes, IE is usually supposed to hold specifically for indicative conditionals.

The distinction between subjunctive and indicative conditionals is often sought in grammatical formulation; it does not show from the formula. But in the case of  $B \rightarrow (A \rightarrow \neg B)$  I think the formula does suggest a grammatical choice. If one assumes  $B$ , then doesn't talking about  $\neg B$  imply that one is assuming the subjunctive mood? What other reason could one have for talking about  $\neg B$ ?

As it stands this argument is still weak; that it is more natural to understand ( $A \rightarrow \neg B$ ) in the subjunctive mood within the conditional  $B \rightarrow (A \rightarrow \neg B)$  is not a reason to conclude that one cannot understand it in the indicative mood. Blackburn argues that even in the indicative sense one may argue that  $B \rightarrow (A \rightarrow B)$  may sometimes be false and thus he seems to advocate against IE (Blackburn, 1994, p. 220-225). However, like me, he does accept that IE does hold in many other cases. This brings forth a difficulty I want to avoid as much as possible, namely, it raises the question under which conditions exactly does IE hold? The rule is weird when it

comes to conspicuous situations, but I don't know what makes a situation conspicuous in the first place, nor do I have a clear understanding of what, for example,  $B \rightarrow (A \rightarrow B)$  *does* mean, if not  $(B \wedge A) \rightarrow B$ . Again, this indicates that intuitions on nested conditionals are poor, and that 'something' is going on that is most likely neglected in the theorems proven above.

To get more grip on the situation consider as another example the (indicative) conditional "If it rains, the match will be canceled." This conditional suggests a causal connection between the weather and the scheduling of the match. But now consider "If the match will be canceled then, if it rains, the match will be canceled." Still in the indicative mood this seems a triviality: if the match is canceled, the match is canceled (no matter if it rains or not). But the "no matter if"-clause does indicate something important. In this case I no longer care for any possible causal connection between the weather and the scheduling. Moreover, I do not find it plausible that just from "the match will be canceled" I can infer a causal connection between the two. This is even more obvious if "if it rains" is replaced by "if I'm wearing red pants".

The line of reasoning adopted here is reminiscent of McGee's counter examples to MP (McGee, 1985). Indeed, it seems weird that one can just infer in general from "it rains" that "if I'm wearing red pants, then it rains". However, I don't think that a failure of MP is the right conclusion here. In stead, I think that the  $A \rightarrow B$  that one can conclude from the conjunction of  $B \rightarrow (A \rightarrow B)$  with  $B$  is not the same as the  $A \rightarrow B$  one started out with. The first is concluded under the supposition that  $B$ , whereas the second would hold even if one is uncertain about  $B$ .

Note that this argument completely follows from taking the Ramsey test seriously. Indeed, in evaluating the truth of a right-nested conditional  $A \rightarrow (B \rightarrow C)$  one first has to add  $A$  to the stock of knowledge and argue on that basis whether or not  $B \rightarrow C$ . This then involves also adding  $B$  to the stock of knowledge and arguing on that basis whether or not  $C$ . The evaluation of  $B \rightarrow C$  is 'tainted' by the assumption that  $A$ . So the Ramsey test suggests both IE and the context-sensitivity of the conditional. What is required now is firstly a notation that will help to illuminate the context-sensitivity of the conditional. I propose the following rules of translation:

$$A \rightarrow B \mapsto A \rightarrow_{\mathfrak{C}} B, \quad (27a)$$

$$A \rightarrow (B \rightarrow C) \mapsto A \rightarrow_{\mathfrak{C}} (B \rightarrow_{\mathfrak{C}_A} C), \quad (27b)$$

$$(A \rightarrow B) \rightarrow C \mapsto (A \rightarrow_{\mathfrak{C}} B) \rightarrow_{\mathfrak{C}} C, \quad (27c)$$

where  $\mathfrak{C}$  denotes the original context, and  $\mathfrak{C}_A$  the context after adding  $A$  to it. These rules elucidate where triviality proofs may be mistaken, and it helps in clarifying my first objection. But also assumptions on the rules for conditionals are to be treated carefully when expressed in this new notation. For example, IE now reads

$$A \rightarrow_{\mathfrak{C}} (B \rightarrow_{\mathfrak{C}_A} C) = (A \wedge B) \rightarrow_{\mathfrak{C}} C = B \rightarrow_{\mathfrak{C}} (A \rightarrow_{\mathfrak{C}_B} C). \quad (28)$$

To clarify my objection 1) I also introduce a notation for the probability functions in which the context appears explicit:

$$P[A] \mapsto P_{\mathfrak{C}}[A], \quad (29a)$$

$$P[B|A] \mapsto P_{\mathfrak{C}}[B|A] \equiv P_{\mathfrak{C}_A}[B]. \quad (29b)$$

Now the thesis can be reformulated as

$$P_{\mathfrak{C}}[A \rightarrow_{\mathfrak{C}} B] = P_{\mathfrak{C}}[B|A]. \quad (30)$$

With this notation the notorious step in Lewis' proof becomes

$$P_{\mathfrak{C}}[A \rightarrow_{\mathfrak{C}} B|B] = P_{\mathfrak{C}_B}[A \rightarrow_{\mathfrak{C}} B] = P_{\mathfrak{C}_B}[A \rightarrow_{\mathfrak{C}_B} B] = P_{\mathfrak{C}_B}[B|A] = P_{\mathfrak{C}}[B|A \wedge B]. \quad (31)$$

Objection 1) is now simply a denial of the second equality sign.

Similarly, the objected step in the proof of theorem 2 now reads

$$\begin{aligned} P_{\mathfrak{C}}[A \rightarrow_{\mathfrak{C}} B|B] &= P_{\mathfrak{C}}[B \rightarrow_{\mathfrak{C}} (A \rightarrow_{\mathfrak{C}} B)] \\ &= P_{\mathfrak{C}}[B \rightarrow_{\mathfrak{C}} (A \rightarrow_{\mathfrak{C}_B} B)] = P_{\mathfrak{C}}[(B \wedge A) \rightarrow_{\mathfrak{C}} B] = P_{\mathfrak{C}}[B|A \wedge B]. \end{aligned} \quad (32)$$

Again, the second equality sign is the troublesome one. Also the proofs for theorems 3 and 4 break down when cleaning them up with the notation of (27). For example, in the proof of theorem 4 context-insensitivity together with MP is used to conclude  $A \rightarrow_{\mathfrak{C}} B$  from the conjunction of  $A \supset B$  with  $(A \supset B) \rightarrow_{\mathfrak{C}} (A \rightarrow_{\mathfrak{C}(A \supset B)} B)$ .

## 6 Comparison With $CE$

Throughout the analysis I have held on to van Fraassen’s view that there is more to a conditional  $A \rightarrow B$  than just the ‘ingredients’  $A$  and  $B$ . In a certain sense my program is complementary to van Fraassen’s. Whereas his tenability result is a real bottom-up approach, finding a minimal logic in which the thesis can hold, I use a more top-down approach adding constraints obtained from triviality results closing in on a maximal logic so to speak. One is tempted to envisage that at some point the two approaches may meet. However, it should also be noted that there are many differences between van Fraassen’s approach and the one adopted here. In fact, there aren’t many points of agreement besides the introduction of context-sensitivity. This is especially made clear when considering the claim that “the Thesis requires logic  $CE$ ” (van Fraassen, 1976, p. 278) which I will deny later on. But let me first elaborate on what this logic is, and what it implies.

**Definition 1.** A Boolean algebra satisfies the minimal logic  $CE$  if there is a (possibly partial) binary operation  $\rightarrow$  such that

$$(A \rightarrow B) \wedge (A \rightarrow C) = A \rightarrow (B \wedge C), \quad (33a)$$

$$(A \rightarrow B) \vee (A \rightarrow C) = A \rightarrow (B \vee C), \quad (33b)$$

$$A \wedge (A \rightarrow B) = A \wedge B, \quad (33c)$$

$$A \rightarrow A = \top, \quad (33d)$$

whenever defined.

Obviously, the idea is that  $A \rightarrow B$  should be defined for sufficiently many pairs  $(A, B)$ . For example, one may demand that the domain should at least include everything that doesn’t lie in the range of the operation. That is,  $A \rightarrow B$  should at least be defined whenever  $A$  and  $B$  themselves are not of the form  $C \rightarrow D$ .

Using these rules, van Fraassen derived that

$$\begin{aligned} P[A \rightarrow B|A] &= \frac{P[(A \rightarrow B) \wedge A]}{P[A]} = \frac{P[A \wedge B]}{P[A]} \\ &= P[B|A] = P[A \rightarrow B]. \end{aligned} \quad (34)$$

And he argues the following:

“So the agreement, by all concerned, about the Thesis is that it is equivalent to the stochastic independence between the conditional and its antecedent. This might provide a new fulcrum for the application of a philosophical critique or defence of the Thesis.” (van Fraassen, 1976, p. 278)

It is surprising that even though van Fraassen himself pointed out a weak link in his tenability result, it has only recently been criticized. Douven and Dietz noticed that this has as a consequence that

$$P[A|A \rightarrow B] = \frac{P[A \wedge (A \rightarrow B)]}{P[A \rightarrow B]} = \frac{P[A \rightarrow B|A]P[A]}{P[A \rightarrow B]} = P[A], \quad (35)$$

and they argue that in some cases this is untenable (Douven & Dietz, 2011, p. 33).

It may be noted that this argument even works when explicitly keeping track of the context in the notation and so the question arises if the minimal logic  $CE$  is really required by the thesis. Fortunately, it isn’t. Van Fraassen derives the rules (33) from the thesis together with three additional assumptions:

1. If two propositions must always have the same probability, then they are identical.
2. If  $A$  is not the empty proposition then  $A \rightarrow B$  and  $A \rightarrow C$  are disjoint if  $B$  and  $C$  are disjoint.
3.  $(A \rightarrow B) \wedge A = A \wedge B$ .

Now I fully agree with the first two assumptions, but am skeptical about the third. In one direction this is just MP with which I agree. More explicitly, I think that for every context  $\mathfrak{C}$  one has

$$(A \rightarrow_{\mathfrak{C}} B) \wedge A \leq A \wedge B. \quad (36)$$

But if I'm deducing a conditional from some proposition, this conditional is phrased in a context in which at least this proposition is true. So, at best, I would be willing to accept for every context  $\mathfrak{C}$  that

$$A \wedge B \leq (A \rightarrow_{\mathfrak{C}, (A \wedge B)} B) \wedge A. \quad (37)$$

In fact, this result *does* follow from the thesis. If a context  $\mathfrak{C}$  is such that  $P_{\mathfrak{C}}[A \wedge B] = 1$ , then also  $P_{\mathfrak{C}}[A] = 1$  and  $P_{\mathfrak{C}}[A \rightarrow_{\mathfrak{C}} B] = P_{\mathfrak{C}}[B|A] = 1$ .

Assuming IE there is also an other version that can be derived. For any context  $\mathfrak{C}$  one has

$$\begin{aligned} P_{\mathfrak{C}}[A \wedge B] &= P_{\mathfrak{C}}[B|A]P_{\mathfrak{C}}[A] = P_{\mathfrak{C}}[A \rightarrow_{\mathfrak{C}} B]P_{\mathfrak{C}}[A] = P_{\mathfrak{C}}[(A \wedge A) \rightarrow_{\mathfrak{C}} B]P_{\mathfrak{C}}[A] \\ &= P_{\mathfrak{C}}[A \rightarrow_{\mathfrak{C}} (A \rightarrow_{\mathfrak{C}, A} B)]P_{\mathfrak{C}}[A] = P_{\mathfrak{C}}[A \rightarrow_{\mathfrak{C}, A} B|A]P_{\mathfrak{C}}[A] = P_{\mathfrak{C}}[(A \rightarrow_{\mathfrak{C}, A} B) \wedge A] \end{aligned} \quad (38)$$

But none of these results come close to validating the triviality result (35).

## 7 Defending Context-Sensitivity

The previous discussion makes clear that what remains of the thesis seems to have only small scope. In general, it does not help one to decide the probability of a conditional, i.e. the thesis has nothing to say about

$$P_{\mathfrak{C}}[A \rightarrow_{\mathfrak{C}'} B] \quad (39)$$

when  $\mathfrak{C} \neq \mathfrak{C}'$ . It appears as if someone adopting the context  $\mathfrak{C}$  has nothing sensible to say about the conditional posed in any other context. This is of course reminiscent of Lewis' original objection against allowing changes in the meaning of the conditional:

“presumably our indicative conditional has a fixed interpretation, the same for speakers with different beliefs, and for one speaker before and after a change in his beliefs. Else how are disagreements about a conditional possible, or changes of mind?” (Lewis, 1976, p. 301)

This calls for a deeper analysis of what is precisely meant by the context-dependence of the conditional.

First of all, I believe it to be uncontroversial that degrees of belief are context-dependent. It thus makes sense to explicitly mention the context when specifying a probability function like  $P_{\mathfrak{C}}$ . It is also sensible to assume that, in general, different persons have different degrees of belief stemming from different backgrounds of knowledge. So in general, when two people are arguing, they use arguments based on different probability functions  $P_{\mathfrak{C}}$ ,  $P_{\mathfrak{C}'}$ . In fact, discussions tend to be quite boring when all participants agree on everything.

According to Lewis, although people may have different states of belief, they can still argue about a proposition like  $A \wedge B$  because  $\wedge$  has a fixed interpretation. However, in general they will assign different degrees of belief to this proposition:  $P_{\mathfrak{C}}[A \wedge B] \neq P_{\mathfrak{C}'}[A \wedge B]$ . How is one then justified in saying that they mean the same thing? It seems that this can be justified by the observation that both evaluate the truth of  $A \wedge B$  in the same way;  $A \wedge B$  is true iff  $A$  is true and  $B$  is true. So if they have the same interpretation for  $A$  and for  $B$ , the fact that  $\wedge$  is truth-functional

seems to warrant that they must have the same interpretation for  $A \wedge B$ . The context-dependence of the conditional may then be seen as a consequence of the failure of truth-functionality.

I do think that the conditional not being truth-functional plays a large part in the difficulties with context-dependence for conditionals. But to get a better view on this, let me first point out a problem with the line of reasoning above when considering the full scope of the belief states. Although it is sufficient to know the truth values of  $A$  and  $B$  to determine the truth value of  $A \wedge B$ , it is not sufficient to know ones degrees of belief  $P_{\mathfrak{C}}[A]$  and  $P_{\mathfrak{C}}[B]$  to determine  $P_{\mathfrak{C}}[A \wedge B]$ . To determine ones degree of belief in  $A \wedge B$ , one has to not only consider  $A$  and  $B$  independently, but also investigate how the two are related. The fact that one can rationally attach different degrees of belief to the conjunction while keeping the degrees of belief in each of the conjuncts fixed indicates that there *is* a context-sensitivity for the conjunction. Still, this does not withhold people from arguing meaningfully about conjunctions. Suppose one person reasons from a context  $\mathfrak{C}$  and another from  $\mathfrak{C}'$  and suppose  $P_{\mathfrak{C}}[A] = P_{\mathfrak{C}'}[A]$ ,  $P_{\mathfrak{C}}[B] = P_{\mathfrak{C}'}[B]$  but  $P_{\mathfrak{C}}[A \wedge B] \neq P_{\mathfrak{C}'}[A \wedge B]$ . The discussion between the two will most likely focus on what precisely is the connection between  $A$  and  $B$ . But the discussion is not precisely meaningful because they mean exactly the same, but because they are able to imagine each others positions.

Actually, this doesn't look like a real difference in the meaning of the logical connective  $\wedge$ , but it does show that there is a sensitivity to the context involved when arguing about propositions formed using the connective. My opinion is that the context-sensitivity for the conditional is much like this. This poses the following question: "Why is there this need to make the context explicit in the specification for the conditional and not for, say, the conjunction?" I think this has to do with the clear intuitions we have about the meaning of conjunctions and disjunctions, and the poor intuition about conditionals. The entire development of probability theory concerns considerations about the first two, but not about the third. For example, one of the axioms is

$$P[A \vee B] = P[A] + P[B] - P[A \wedge B], \quad (40)$$

but none of the axioms involve conditionals. Consequently, probability theory is respectful for the meaning of disjunctions and conjunctions, but not necessarily for conditionals. In this sense, I see the thesis as an attempt to extend the set of axioms for probability theory to make it also respectful for conditionals. But this must be done carefully, and known results have to be revised in this spirit. In particular, whereas the definition

$$P[B|A] := \frac{P[A \wedge B]}{P[A]} \quad (41)$$

of conditional probability does give a reasonable idea for updating beliefs about non-conditional propositions upon learning  $A$ , it is not to be suspected that this rule is also satisfactory for conditionals. That is, it is respectful to the meaning-shifts occurring in the connectives  $\vee$  and  $\wedge$  upon learning  $A$ , but it isn't for the meaning-shift occurring in the conditional. And that is why the explicit mentioning of the context is required for conditionals, but not for other connectives.

All in all I think that the context-sensitivity of the meaning of conditionals will play a great part in the better understanding of conditionals in general. Although this idea is not new, it still remains to be further investigated how the meaning of a conditional may shift as the context shifts. That is, it has to be made more explicit what exactly the difference is between for example  $\rightarrow_{\mathfrak{C}}$  and  $\rightarrow_{\mathfrak{C}_A}$ , which obviously starts with making explicit what signifies the difference between  $\mathfrak{C}$  and  $\mathfrak{C}_A$ . Also, for left-nested conditionals it has to be explained what it means to add a conditional to a stock of beliefs. Another question is whether or not in this setting the thesis can also be extended to also cover nested conditionals. As a specific task, one may hope to find a new definition of conditional probability that respects all logical connectives so that one may say that

$$P[A \rightarrow B] = P[B \uparrow A] \quad (42)$$

for all propositions  $A$  and  $B$  such that  $P[B \uparrow A] = P[B|A]$  whenever  $A$  and  $B$  are themselves not conditionals. This is something I hope to be elaborating on in a future article.

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